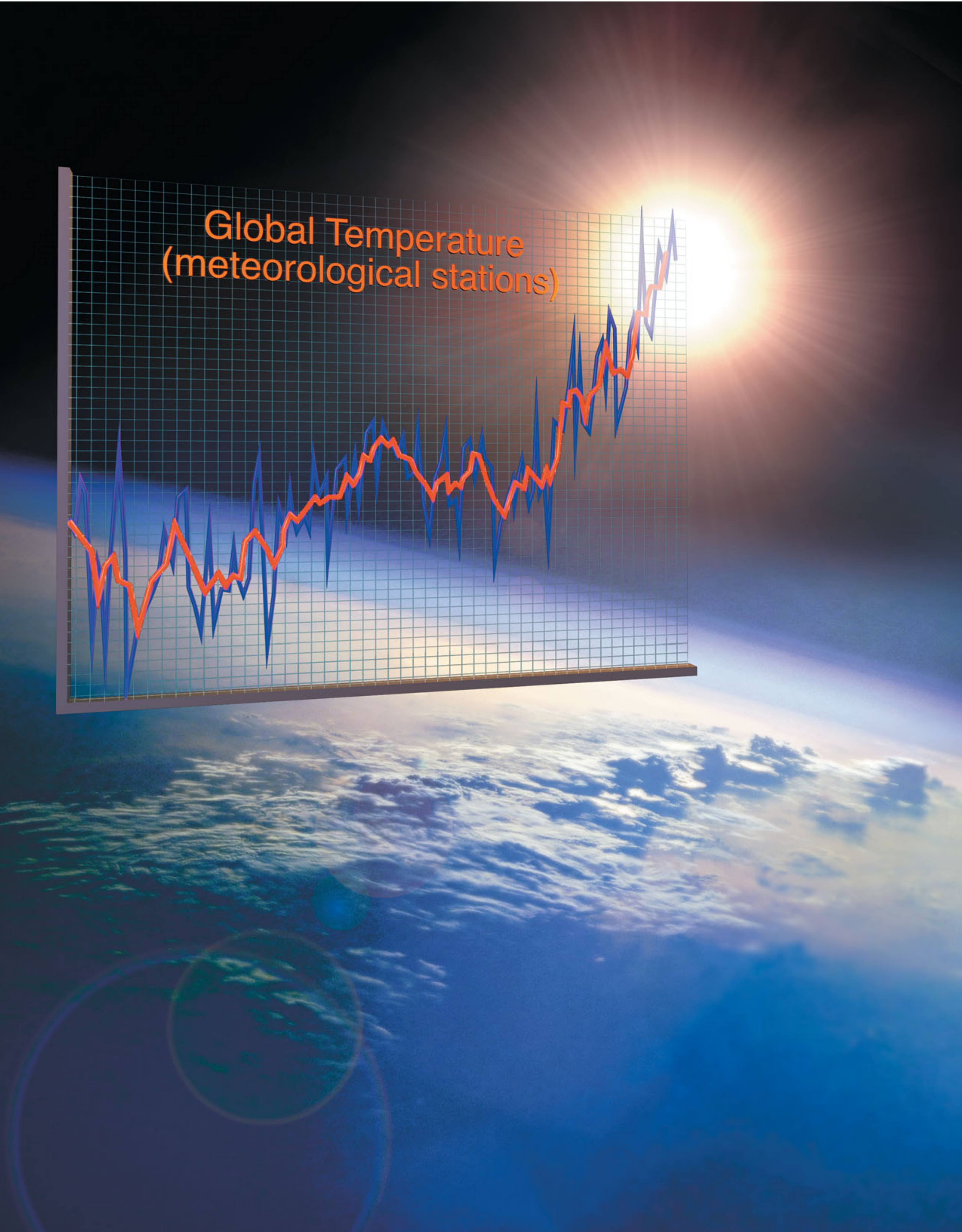

Global Temperature
(meteorological stations)

CHAPTER 2

Measurement and Problem Solving

"The important thing in science is not so much to obtain new facts as to discover new ways of thinking about them."

SIR WILLIAM LAWRENCE BRAGG (1890–1971)

2.1 Measuring Global Temperatures

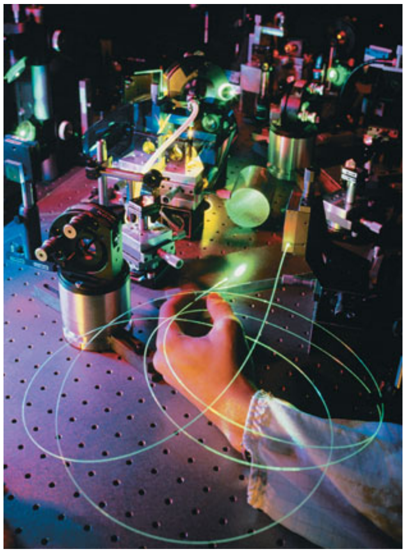
Global warming has become a household term. Average global temperatures affect things from agriculture to weather and ocean levels. The media report that global temperatures are increasing. These reports are based on the work of scientists who—after analyzing records from thousands of temperature-measuring stations around the world—concluded that average global temperatures have risen by 0.6 °C in the last century.

A unit is a standard, agreed-on quantity by which other quantities are measured.

Notice how the scientists reported their results. What if they had reported a temperature increase of simply 0.6 without any *units*? The result would be unclear. Units are extremely important in reporting and working with scientific measurements, and they must always be included. Suppose that the scientists had included additional zeros in their results—for example, 0.60 °C or 0.600 °C—or that they had reported the number their computer displayed after averaging many measurements, something like 0.58759824 °C. Would these convey the same information? Not really. Scientists agree to a standard way of reporting measured quantities in which the number of reported digits reflects the precision in the measurement—more digits, more precision; fewer digits, less precision. Numbers are usually written so that the uncertainty is indicated by the last reported digit. For example, by reporting a temperature increase of 0.6 °C, the scientists mean 0.6 ± 0.1 °C (± means plus or minus). The temperature rise could be as much as 0.7 °C or as little as 0.5 °C, but it is not 1.0 °C. The degree of certainty in this particular measurement is critical, influencing political decisions that directly affect people's lives.

◀ The graph in this image displays average global temperatures (relative to the mean) over the past 100 years.

2.2 Scientific Notation: Writing Large and Small Numbers



▲ Lasers such as this one can measure time periods as short as 1×10^{-15} s.

Science has constantly pushed the boundaries of the very large and the very small. We can, for example, now measure time periods as short as 0.000000000000001 seconds and distances as great as 14,000,000,000 light-years. Because the many zeros in these numbers are cumbersome to write, scientists use **scientific notation** to write them more compactly. In scientific notation, 0.000000000000001 is 1×10^{-15}, and 14,000,000,000 is 1.4×10^{10}. A number written in scientific notation consists of a **decimal part,** a number that is usually between 1 and 10, and an **exponential part,** 10 raised to an **exponent,** *n*.

$$\underbrace{1.2}_{\text{decimal part}} \times \underbrace{10^{-10}}_{\text{exponential part}} \leftarrow \text{exponent } (n)$$

A positive exponent means 1 multiplied by 10 *n* times.

$$10^0 = 1$$
$$10^1 = 1 \times 10 = 10$$
$$10^2 = 1 \times 10 \times 10 = 100$$
$$10^3 = 1 \times 10 \times 10 \times 10 = 1000$$

A negative exponent ($-n$) means 1 divided by 10 *n* times.

$$10^{-1} = \frac{1}{10} = 0.1$$
$$10^{-2} = \frac{1}{10 \times 10} = 0.01$$
$$10^{-3} = \frac{1}{10 \times 10 \times 10} = 0.001$$

To convert a number to scientific notation, move the decimal point (either to the left or to the right, as needed) to obtain a number between 1 and 10 and then multiply that number (the decimal part) by 10 raised to the power that reflects the movement of the decimal point. For example, to write 5983 in scientific notation, move the decimal point to the left three places to get 5.983 (a number between 1 and 10) and then multiply the decimal part by 1000 to compensate for moving the decimal point.

$$5983 = 5.983 \times 1000$$

1000 is 10^3

$$= 5.983 \times 10^3$$

You can do this in one step by counting how many places you move the decimal point to obtain a number between 1 and 10 and then writing the decimal part multiplied by 10 raised to the number of places you moved the decimal point.

$$5983 = 5.983 \times 10^3$$

3 2 1

If the decimal point is moved to the left, as in the previous example, the exponent is positive. If the decimal is moved to the right, the exponent is negative.

$$0.00034 = 3.4 \times 10^{-4}$$

1 2 3 4

To express a number in scientific notation:

1. Move the decimal point to obtain a number between 1 and 10.
2. Write the result from Step 1 multiplied by 10 raised to the number of places you moved the decimal point.
   - The exponent is positive if you moved the decimal point to the left.
   - The exponent is negative if you moved the decimal point to the right.

EXAMPLE 2.1 Scientific Notation

The 2010 U.S. population was estimated to be 308,255,000 people. Express this number in scientific notation.

To obtain a number between 1 and 10, move the decimal point to the left 8 decimal places; the exponent is 8. Since you move the decimal point to the left, the sign of the exponent is positive.

SOLUTION

$308{,}255{,}000 \text{ people} = 3.08255 \times 10^{8} \text{ people}$

▶SKILLBUILDER 2.1 | Scientific Notation

The total U.S national debt in 2010 was approximately $12,102,000,000,000. Express this number in scientific notation.

Note: The answers to all Skillbuilders appear at the end of the chapter.

▶FOR MORE PRACTICE Example 2.18; Problems 31, 32.

EXAMPLE 2.2 Scientific Notation

The radius of a carbon atom is approximately 0.000000000070 m. Express this number in scientific notation.

To obtain a number between 1 and 10, move the decimal point to the right 11 decimal places; therefore, the exponent is 11. Since the decimal point was moved to the right, the sign of the exponent is negative.

SOLUTION

$0.000000000070 \text{ m} = 7.0 \times 10^{-11} \text{ m}$

▶SKILLBUILDER 2.2 | Scientific Notation

Express the number 0.000038 in scientific notation.

▶FOR MORE PRACTICE Problems 33, 34.

CONCEPTUAL CHECKPOINT 2.1

The radius of a dust speck is 4.5×10^{-3} mm. What is the correct value of this number in decimal notation (i.e., express the number without using scientific notation)?

(a) 4500 mm

(b) 0.045 mm

(c) 0.0045 mm

(d) 0.00045 mm

Note: The answers to all Conceptual Checkpoints appear at the end of the chapter.

2.3 Significant Figures: Writing Numbers to Reflect Precision

▲ Since pennies come in whole numbers, 7 pennies means 7.00000… pennies. This is an exact number and therefore never limits significant figures in calculations.

If we tell someone we have seven pennies, our meaning is clear. Pennies come in whole numbers, and seven pennies means seven whole pennies—it is unlikely that we would have 7.4 pennies. However, if we tell someone that we have a 10-g gold bar, the meaning is *unclear.* Our knowledge of the actual amount of gold in the bar depends on how precisely it was measured, which in turn depends on the scale or balance used to make the measurement. As we just learned, measured quantities are written to reflect the uncertainty in the measurement. If the gold measurement was rough, we could describe the bar as containing "10 g of gold." If a more precise balance was used, we could write the gold content as "10.0 g." We would report an even more precise measurement as "10.00 g."

Scientific numbers are reported so that every digit is certain except the last, which is estimated. For example, suppose a reported measurement is:

$$\underbrace{45.87}_{\textbf{certain}}\underbrace{\mathit{2}}_{\textit{estimated}}$$

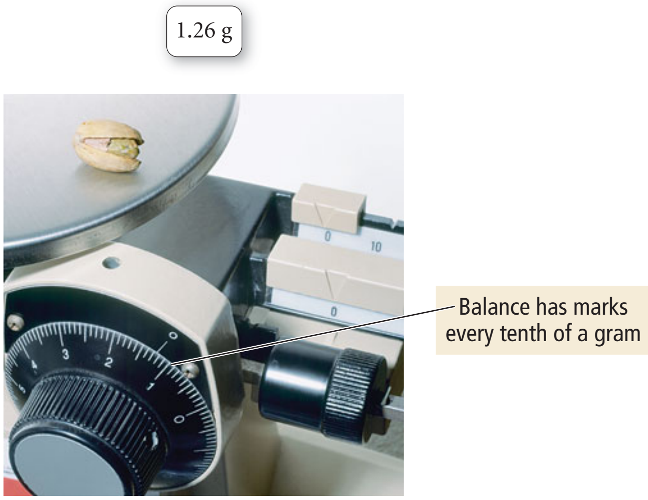
The first four digits are certain; the last digit is estimated.

▲ Our knowledge of the amount of gold in a 10-g gold bar depends on how precisely it was measured.

Suppose that we weigh an object on a balance with marks at every 1 g, and the pointer is between the 1-g mark and the 2-g mark (▼ Figure 2.1) but much closer to the 1-g mark. To record the measurement, we mentally divide the space between the 1- and 2-g marks into 10 equal spaces and estimate the position of the pointer. In this case, the pointer indicates about 1.2 g. We then write the measurement as 1.2 g, indicating that we are sure of the "1" but have estimated the ".2."

If we measure the same object using a balance with marks every tenth of a gram, we need to write the result with more digits. For example, suppose that on this more precise balance the pointer is between the 1.2-g mark and the 1.3-g mark (▼ Figure 2.2). We again divide the space between the two marks into 10 equal spaces and estimate the third digit. In the case of the nut shown in Figure 2.2, we report 1.26 g. Digital balances usually have readouts that report the mass to the correct number of digits.

▲ **FIGURE 2.1 Estimating tenths of a gram** This balance has markings every 1 g, so we estimate to the tenths place. To estimate between markings, mentally divide the space into 10 equal spaces and estimate the last digit. This reading is 1.2 g.



▲ **FIGURE 2.2 Estimating hundredths of a gram** Since this scale has markings every 0.1 g, we estimate to the hundredths place. The correct reading is 1.26 g.

EXAMPLE 2.3 Reporting the Right Number of Digits

The bathroom scale in ▼ Figure 2.3 has markings at every 1 lb. Report the reading to the correct number of digits.

▲ **FIGURE 2.3 Reading a bathroom scale**

SOLUTION

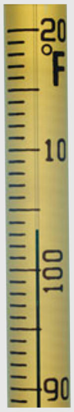
Since the pointer is between the 147- and 148-lb markings, mentally divide the space between the markings into 10 equal spaces and estimate the next digit. In this case, the result should be reported as:

147.7 lb

What if you estimated a little differently and wrote 147.6 lb? In general, one unit of difference in the last digit is acceptable because the last digit is estimated and different people might estimate it slightly differently. However, if you wrote 147.2 lb, you would clearly be wrong.

▶**SKILLBUILDER 2.3 | Reporting the Right Number of Digits**

A thermometer is used to measure the temperature of a backyard hot tub, and the reading is shown in ▼ Figure 2.4. Write the temperature reading to the correct number of digits.



◀ **FIGURE 2.4 Reading a thermometer**

▶**FOR MORE PRACTICE** Example 2.19; Problems 41, 42.

COUNTING SIGNIFICANT FIGURES

The non–place-holding digits in a measurement are **significant figures** (or **significant digits**) and, as we have seen, represent the precision of a measured quantity. The greater the number of significant figures, the greater the precision of the measurement. We can determine the number of significant figures in a written number fairly easily; however, if the number contains zeros, we must distinguish between the zeros that are significant and those that simply mark the decimal place. In the number 0.002, for example, the leading zeros simply mark the decimal place; they *do not* add to the precision of the measurement. In the number 0.00200, the trailing zeros *do* add to the precision of the measurement.

To determine the number of significant figures in a number, follow these rules:

1. All nonzero digits are significant.

   1.05 0.0110

2. Interior zeros (zeros between two numbers) are significant.

   4.0208 50.1

3. Trailing zeros (zeros to the right of a nonzero number) that fall after a decimal point are significant.

   5.10 3.00

4. Trailing zeros that fall before a decimal point are significant.

50.00 1700.24

5. Leading zeros (zeros to the left of the first nonzero number) are not significant. They only serve to locate the decimal point.

For example, the number 0.0005 has only one significant digit.

6. **Trailing** zeros at the end of a number, but before an *implied* decimal point, are ambiguous and should be avoided by using scientific notation.

For example, it is unclear if the number 350 has two or three significant figures. We can avoid confusion by writing the number as 3.5×10^2 to indicate two significant figures or as 3.50×10^2 to indicate three.

When a number is expressed in scientific notation, all trailing zeros are significant.

Some books put a decimal point after one or more trailing zeros if the zeros are to be considered significant. We avoid that practice in this book, but you should be aware of it.

EXACT NUMBERS

Exact numbers have an unlimited number of significant figures. Exact numbers originate from three sources:

- Exact counting of discrete objects. For example, 3 atoms means 3.00000…atoms.
- *Defined quantities*, such as the number of centimeters in 1 m. Because 100 cm is defined as 1 m,

$$100\,\text{cm} = 1\,\text{m} \text{ means } 100.00000\ldots\,\text{cm} = 1.0000000\ldots\,\text{m}$$

  Note that some conversion factors are defined quantities whereas others are not.
- Integral numbers that are part of an equation. For example, in the equation, $radius = \frac{diameter}{2}$, the number 2 is exact and therefore has an unlimited number of significant figures.

EXAMPLE 2.4 Determining the Number of Significant Figures in a Number

How many significant figures are in each number?

(a) 0.0035
(b) 1.080
(c) 2371
(d) 2.97×10^5
(e) 1 dozen = 12
(f) 100.00
(g) 100,000

	SOLUTION
The 3 and the 5 are significant (rule 1). The leading zeros only mark the decimal place and are not significant (rule 5).	**(a)** 0.0035 two significant figures
The interior zero is significant (rule 2), and the trailing zero is significant (rule 3). The 1 and the 8 are also significant (rule 1).	**(b)** 1.080 four significant figures
All digits are significant (rule 1).	**(c)** 2371 four significant figures
All digits in the decimal part are significant (rule 1).	**(d)** 2.97×10^5 three significant figures
Defined numbers are exact and therefore have an unlimited number of significant figures.	**(e)** 1 dozen = 12 unlimited significant figures
The 1 is significant (rule 1), and the trailing zeros before the decimal point are significant (rule 4). The trailing zeros after the decimal point are also significant (rule 3).	**(f)** 100.00 five significant figures
This number is ambiguous. Write as 1×10^5 to indicate one significant figure or as 1.00000×10^5 to indicate six significant figures.	**(g)** 100,000 ambiguous

▶SKILLBUILDER 2.4 | Determining the Number of Significant Figures in a Number

How many significant figures are in each number?

(a) 58.31
(b) 0.00250
(c) 2.7×10^3
(d) $1\,\text{cm} = 0.01\,\text{m}$
(e) 0.500
(f) 2100

▶FOR MORE PRACTICE Example 2.20; Problems 43, 44, 45, 46, 47, 48.

CONCEPTUAL CHECKPOINT 2.2

A researcher reports that the Spirit rover on the surface of Mars recently measured the temperature to be −25.49 °F. What is the actual temperature?

(a) between −25.490 °F and −25.499 °F
(b) between −25.48 °F and −25.50 °F
(c) between −25.4 °F and −25.5 °F
(d) exactly −25.49 °F

2.4 Significant Figures in Calculations

When we use measured quantities in calculations, the results of the calculation must reflect the precision of the measured quantities. We should not lose or gain precision during mathematical operations.

MULTIPLICATION AND DIVISION

In multiplication or division, the result carries the same number of significant figures as the factor with the fewest significant figures.

For example:

$$\underset{\text{(3 sig. figures)}}{5.02} \times \underset{\text{(5 sig. figures)}}{89.665} \times \underset{\text{(2 sig. figures)}}{0.10} = 45.0118 = \underset{\text{(2 sig. figures)}}{45}$$

The intermediate result (in blue) is rounded to two significant figures to reflect the least precisely known factor (0.10), which has two significant figures.

In division, we follow the same rule.

$$\underset{\text{(4 sig. figures)}}{5.892} \div \underset{\text{(3 sig. figures)}}{6.10} = 0.96590 = \underset{\text{(3 sig. figures)}}{0.966}$$

The intermediate result (in blue) is rounded to three significant figures to reflect the least precisely known factor (6.10), which has three significant figures.

ROUNDING

When we round to the correct number of significant figures:

we round down if the last (or leftmost) digit dropped is 4 or less;
we round up if the last (or leftmost) digit dropped is 5 or more.

CHEMISTRY IN THE MEDIA

The COBE Satellite and Very Precise Measurements That Illuminate Our Cosmic Past

Since the earliest times, humans have wondered about the origins of our planet. Science has slowly probed this question and has developed theories for how the universe and the Earth began. The most accepted theory today about the origin of the universe is the Big Bang theory. According to the Big Bang theory, the universe began in a tremendous expansion about 13.7 billion years ago and has been expanding ever since. A measurable prediction of this theory is the presence of a remnant "background radiation" from the expansion of the universe. That remnant is characteristic of the current temperature of the universe. When the Big Bang occurred, the temperature of the universe was very hot and the associated radiation very bright. Today, 13.7 billion years later, the temperature of the universe is very cold and the background radiation very faint.

In the early 1960s, Robert H. Dicke, P. J. E. Peebles, and their coworkers at Princeton University began to build a device to measure this background radiation and thus take a direct look into the cosmological past and provide evidence for the Big Bang theory. At about the same time, quite by accident, Arno Penzias and Robert Wilson of Bell Telephone Laboratories measured excess radio noise on one of their communications satellites. As it turned out, this noise was the background radiation that the Princeton scientists were looking for. The two groups published papers together in 1965 reporting their findings along with the corresponding current temperature of the universe, about 3 degrees above absolute zero, or 3 K. We will define temperature measurement scales in Chapter 3. For now, know that 3 K is an extremely low temperature (460 degrees below zero on the Fahrenheit scale).

In 1989, the Cosmic Background Explorer (COBE) satellite was developed by NASA's Goddard Space Flight Center to measure the background radiation more precisely. The COBE satellite determined that the background radiation corresponded to a universe with a temperature of 2.735 K. (Notice the difference in significant figures from the previous measurement.) It went on to measure tiny fluctuations in the background radiation that amount to temperature differences of 1 part in 100,000. These fluctuations, though small, are an important prediction of the Big Bang theory. Scientists announced that the COBE satellite had produced the strongest evidence yet for the Big Bang theory of the creation of the universe. This is the way that science works. Measurement, and precision in measurement, are important to understanding the world—so important that we dedicate most of this chapter just to the concept of measurement.

CAN YOU ANSWER THIS? *How many significant figures are there in each of the preceding temperature measurements (3 K, 2.735 K)?*

▲ The COBE Satellite, launched in 1989 to measure background radiation. Background radiation is a remnant of the Big Bang—the expansion that is believed to have formed the universe.

Consider rounding each of these numbers to two significant figures.

2.33 rounds to 2.3
2.37 rounds to 2.4
2.34 rounds to 2.3
2.35 rounds to 2.4

We use only the *last (or leftmost) digit being dropped* to decide in which direction to round—we ignore all digits to the right of it. For example, to round 2.349 to two significant figures, only the 4 in the hundredths place (2.349) determines which direction to round—the 9 is irrelevant.

2.349 rounds to 2.3

For calculations involving multiple steps, we round only the final answer—we do not round off between steps. This prevents small rounding errors from affecting the final answer.

EXAMPLE 2.5 Significant Figures in Multiplication and Division

Perform each calculation to the correct number of significant figures.

(a) $1.01 \times 0.12 \times 53.51 \div 96$
(b) $56.55 \times 0.920 \div 34.2585$

	SOLUTION
Round the intermediate result (in blue) to two significant figures to reflect the two significant figures in the least precisely known quantities (0.12 and 96).	**(a)** $1.01 \times 0.12 \times 53.51 \div 96 = 0.067556 = 0.068$
Round the intermediate result (in blue) to three significant figures to reflect the three significant figures in the least precisely known quantity (0.920).	**(b)** $56.55 \times 0.920 \div 34.2585 = 1.51863 = 1.52$

▶SKILLBUILDER 2.5 | Significant Figures in Multiplication and Division

Perform each calculation to the correct number of significant figures.

(a) $1.10 \times 0.512 \times 1.301 \times 0.005 \div 3.4$

(b) $4.562 \times 3.99870 \div 89.5$

▶FOR MORE PRACTICE Examples 2.21, 2.22; Problems 57, 58, 59, 60.

ADDITION AND SUBTRACTION

In addition or subtraction, the result carries the same number of decimal places as the quantity carrying the fewest decimal places.

For example:

$$\begin{array}{r} 5.74 \\ 0.823 \\ +\,2.651 \\ \hline 9.214 \end{array} = 9.21$$

It is sometimes helpful to draw a vertical line directly to the right of the number with the fewest decimal places. The line shows the number of decimal places that should be in the answer.

We round the intermediate answer (in blue) to two decimal places because the quantity with the fewest decimal places (5.74) has two decimal places.

For subtraction, we follow the same rule. For example:

$$\begin{array}{r} 4.8 \\ -\,3.965 \\ \hline 0.835 \end{array} = 0.8$$

We round the intermediate answer (in blue) to one decimal place because the quantity with the fewest decimal places (4.8) has one decimal place. Remember: *For multiplication and division, the quantity with the fewest* ***significant figures*** *determines the number of significant figures in the answer. For addition and subtraction, the quantity with the fewest* ***decimal places*** *determines the number of decimal places in* *the answer.* In multiplication and division we focus on significant figures, but in addition and subtraction we focus on decimal places. When a problem involves addition and subtraction, the answer may have a different number of significant figures than the initial quantities. For example:

$$28.02 - 27.99 = 0.003$$

Initial quantities each have four significant figures

Answer has only one significant figure

The answer has only one significant figure, even though the initial quantities each had four significant figures.

EXAMPLE 2.6 Significant Figures in Addition and Subtraction

Perform the calculations to the correct number of significant figures.

(a)
$$\begin{array}{r} 0.987 \\ +125.1 \\ -1.22 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 0.765 \\ -3.449 \\ -5.98 \\ \hline \end{array}$$

SOLUTION

Round the intermediate answer (in blue) to one decimal place to reflect the quantity with the fewest decimal places (125.1). Notice that 125.1 is not the quantity with the fewest significant figures—it has four while the other quantities only have three—but because it has the fewest decimal places, it determines the number of decimal places in the answer.

(a)
$$\begin{array}{r} 0.987 \\ +125.1 \\ -1.22 \\ \hline 124.867 \end{array} = 124.9$$

Round the intermediate answer (in blue) to two decimal places to reflect the quantity with the fewest decimal places (5.98).

(b)
$$\begin{array}{r} 0.765 \\ -3.449 \\ -5.98 \\ \hline -8.664 \end{array} = -8.66$$

▶SKILLBUILDER 2.6 | Significant Figures in Addition and Subtraction

Perform the calculations to the correct number of significant figures.

(a)
$$\begin{array}{r} 2.18 \\ +5.621 \\ +1.5870 \\ -1.8 \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 7.876 \\ -0.56 \\ +123.792 \\ \hline \end{array}$$

▶**FOR MORE PRACTICE** Example 2.23; Problems 61, 62, 63, 64.

CALCULATIONS INVOLVING BOTH MULTIPLICATION/DIVISION AND ADDITION/SUBTRACTION

In calculations involving both multiplication/division and addition/subtraction, we do the steps in parentheses first; determine the correct number of significant figures in the intermediate answer; then do the remaining steps.

For example:

$$3.489 \times (5.67 - 2.3)$$

We complete the subtraction step first.

$$5.67 - 2.3 = 3.37$$

We use the subtraction rule to determine that the intermediate answer (3.37) has only one significant decimal place. To avoid small errors, it is best not to round at this point; instead, we underline the least significant figure as a reminder.

$$= 3.489 \times 3.\underline{3}7$$

We then do the multiplication step.

$$3.489 \times 3.\underline{3}7 = 11.758 = 12$$

We use the multiplication rule to determine that the intermediate answer (11.758) rounds to two significant figures (12) because it is limited by the two significant figures in $3.\underline{3}7$.

EXAMPLE 2.7 Significant Figures in Calculations Involving Both Multiplication/Division and Addition/Subtraction

Perform the calculations to the correct number of significant figures.

(a) $6.78 \times 5.903 \times (5.489 - 5.01)$
(b) $19.667 - (5.4 \times 0.916)$

SOLUTION

Do the step in parentheses first. Use the subtraction rule to mark 0.479 to two decimal places since 5.01, the number in the parentheses with the least number of decimal places, has two.

(a) $6.78 \times 5.903 \times (5.489 - 5.01)$
$= 6.78 \times 5.903 \times (0.479)$
$= 6.78 \times 5.903 \times 0.4\underline{7}9$

Then perform the multiplication and round the answer to two significant figures since the number with the least number of significant figures has two.

$6.78 \times 5.903 \times 0.4\underline{7}90 = 19.1707$
$= 19$

Do the step in parentheses first. The number with the least number of significant figures within the parentheses (5.4) has two, so mark the answer to two significant figures.

(b) $19.667 - (5.4 \times 0.916)$
$= 19.667 - (4.9464)$
$= 19.667 - 4.\underline{9}464$

Then perform the subtraction and round the answer to one decimal place since the number with the least number of decimal places has one.

$19.667 - 4.\underline{9}464 = 14.7206$
$= 14.7$

▶SKILLBUILDER 2.7 | Significant Figures in Calculations Involving Both Multiplication/Division and Addition/Subtraction

Perform each calculation to the correct number of significant figures.

(a) $3.897 \times (782.3 - 451.88)$

(b) $(4.58 \div 1.239) - 0.578$

▶FOR MORE PRACTICE Example 2.24; Problems 65, 66, 67, 68.

CONCEPTUAL CHECKPOINT 2.3

Which calculation would have its result reported to the *greater* number of significant figures?

(a) $3 + (15/12)$

(b) $(3 + 15)/12$

2.5 The Basic Units of Measurement

By themselves, numbers have limited meaning. Read this sentence: When my son was 7 he walked 3, and when he was 4 he threw his baseball 8 and said his school was 5 away. The sentence is confusing because we don't know what the numbers mean—the **units** are missing. The meaning becomes clear, however, when we add the missing units to the numbers: When my son was 7 *months old* he walked 3 *steps*, and when he was 4 *years old* he threw his baseball 8 *feet* and said his school was 5 *minutes* away. Units make all the difference. In chemistry, units are critical. Never write a number by itself; always use its associated units—otherwise your work will be as confusing as the initial sentence.

The two most common unit systems are the **English system,** used in the United States, and the **metric system,** used in most of the rest of the world. The English system uses units such as inches, yards, and pounds, while the metric system uses centimeters, meters, and kilograms. The most convenient system for science measurements is based on the metric system and is called the **International System** of units or **SI units.** SI units are a set of standard units agreed on by scientists throughout the world.

The abbreviation *SI* comes from the French *le Système International.*

THE STANDARD UNITS

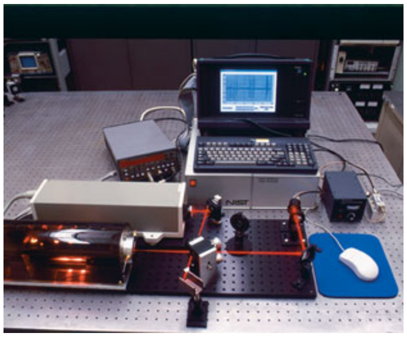
TABLE 2.1 Important SI Standard Units

Quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature*	kelvin	K

*Temperature units are discussed in Chapter 3.

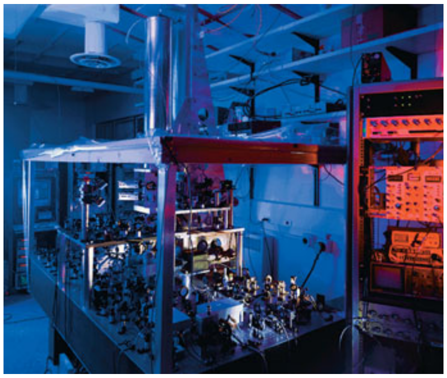
Table 2.1 lists the standard units in the SI system. They include the **meter (m)** as the standard unit of length; the **kilogram (kg)** as the standard unit of mass; and the **second (s)** as the standard unit of time. Each of these standard units is precisely defined. The meter is defined as the distance light travels in a certain period of

▶ Science uses instruments to make measurements. Every instrument is calibrated in a particular unit without which the measurements would be meaningless.



▲ **FIGURE 2.5 The standard of length** The definition of a meter, established by international agreement in 1983, is the distance that light travels in vacuum in 1/299,792,458 s. **Question:** Why is such a precise standard necessary?

▲ **FIGURE 2.6 The standard of mass** A duplicate of the international standard kilogram, called kilogram 20, is kept at the National Institute of Standards and Technology near Washington, DC.



▲ **FIGURE 2.7 The standard of time** The second is defined, using an atomic clock, as the duration of 9,192,631,770 periods of the radiation emitted from a certain transition in a cesium-133 atom.

time: 1/299,792,458 s (▲ Figure 2.5). (The speed of light is 3.0×10^8 m/s.) The kilogram is defined as the mass of a block of metal kept at the International Bureau of Weights and Measures at Sèvres, France (▲ Figure 2.6). The second is defined using an atomic standard (▲ Figure 2.7).

A nickel (5 cents) has a mass of about 5 grams.

Most people are familiar with the SI standard unit of time, the second. However, if you live in the United States, you may be less familiar with the meter and the kilogram. The meter is slightly longer than a yard (a yard is 36 in. while a meter is 39.37 in.). A 100-yd football field measures only 91.4 m.

The kilogram is a measure of mass, which is different from weight. The **mass** of an object is a measure of the quantity of matter within it, whereas the weight of an object is a measure of the gravitational pull on that matter. Consequently, weight depends on gravity while mass does not. If you were to weigh yourself on Mars, for example, the lower gravity would pull you toward the scale less than Earth's gravity would, resulting in a lower weight. A 150-lb person on Earth weighs only 57 lb on Mars. However, the person's mass, the quantity of matter in his or her body, remains the same. A kilogram of mass is the equivalent of 2.205 lb of weight on Earth, so if we express mass in kilograms, a 150-lb person on Earth has a mass of approximately 68 kg. A second common unit of mass is the gram (g), defined as follows:

$$1000\,\text{g} = 10^3\,\text{g} = 1\,\text{kg}$$

PREFIX MULTIPLIERS

The SI system employs **prefix multipliers** (Table 2.2) with the standard units. These multipliers change the value of the unit by powers of 10. For example, the kilometer (km) has the prefix *kilo-*, meaning 1000 or 10^3. Therefore:

$$1\,\text{km} = 1000\,\text{m} = 10^3\,\text{m}$$

Similarly, the millisecond (ms) has the prefix *milli-*, meaning 0.001 or 10^{-3}.

$$1\,\text{ms} = 0.001\,\text{s} = 10^{-3}\,\text{s}$$

▲ The diameter of a quarter is about 2.4 cm. **Question:** Why would you *not* use meters to make this measurement?

TABLE 2.2 SI Prefix Multipliers

Prefix	Symbol	Multiplier	
tera-	T	1,000,000,000,000	(10^{12})
giga-	G	1,000,000,000	(10^{9})
mega-	M	1,000,000	(10^{6})
kilo-	k	1,000	(10^{3})
deci-	d	0.1	(10^{-1})
centi-	c	0.01	(10^{-2})
milli-	m	0.001	(10^{-3})
micro-	μ	0.000001	(10^{-6})
nano-	n	0.000000001	(10^{-9})
pico-	p	0.000000000001	(10^{-12})
femto-	f	0.000000000000001	(10^{-15})

The prefix multipliers allow us to express a wide range of measurements in units that are similar in size to the quantity we are measuring. You should choose the prefix multiplier that is most convenient for a particular measurement. For example, to measure the diameter of a quarter, use centimeters because a quarter has a diameter of about 2.4 cm. A centimeter is a common metric unit and is about equivalent to the width of a pinky finger (2.54 cm = 1 in.). The millimeter could also work to express the diameter of the quarter; then the quarter would measure 24 mm. The kilometer, however, would not work as well since, in that unit, the quarter's diameter is 0.000024 km. Pick a unit similar in size to (or smaller than) the quantity you are measuring. Consider expressing the length of a short chemical bond, about 1.2×10^{-10} m. Which prefix multiplier should you use? The most convenient one is probably the picometer (pico = 10^{-12}). Chemical bonds measure about 120 pm.

TABLE 2.3 Some Common Units and Their Equivalents

Length
1 kilometer (km) = 0.6214 mile (mi)
1 meter (m) = 39.37 inches (in.) = 1.094 yards (yd)
1 foot (ft) = 30.48 centimeters (cm)
1 inch (in.) = 2.54 centimeters (cm) (exact)
Mass
1 kilogram (kg) = 2.205 pounds (lb)
1 pound (lb) = 453.59 grams (g)
1 ounce (oz) = 28.35 grams (g)
Volume
1 liter (L) = 1000 milliliters (mL) = 1000 cubic centimeters (cm^3)
1 liter (L) = 1.057 quarts (qt)
1 U.S. gallon (gal) = 3.785 liters (L)

CONCEPTUAL CHECKPOINT 2.4

What would be the most convenient unit to express the dimensions of a polio virus, which is about 2.8×10^{-8} m in diameter?

(a) Mm

(b) mm

(c) μm

(d) nm

DERIVED UNITS

A derived unit is formed from other units. For example, many units of **volume,** a measure of space, are derived units. Any unit of length, when cubed (raised to the third power), becomes a unit of volume. Therefore, cubic meters (m^3), cubic centimeters (cm^3), and cubic millimeters (mm^3) are all units of volume. In these units, a three-bedroom house has a volume of about 630 m^3, a can of soda pop has a volume of about 350 cm^3, and a rice grain has a volume of about 3 mm^3. We also use the **liter (L)** and milliliter (mL) to express volume (although these are not derived units). A gallon is equal to 3.785 L. A milliliter is equivalent to 1 cm^3. Table 2.3 lists some common units and their equivalents.

2.6 Problem Solving and Unit Conversions

Problem solving is one of the most important skills you will acquire in this course. Not only will this skill help you succeed in chemistry, but it will help you to learn how to think critically, which is important in every area of knowledge. My daughter, a freshman in high school, recently came to me for help on an algebra problem. The statement of the problem went something like this:

> Sam and Sara live 11 miles apart. Sam leaves his house traveling at 6 miles per hour toward Sara's house. Sara leaves her house traveling at 3 miles per hour toward Sam's house. How much time until Sam and Sara meet?

Solving the problem requires setting up the equation $11 - 6t = 3t$. Although my daughter could solve this equation for t quite easily, getting to the equation from the problem statement was another matter—that process requires *critical thinking.* You can't succeed in chemistry—or in life, really—without developing critical thinking skills. Learning how to solve chemical problems will help you develop these kinds of skills.

Although no simple formula applies to every problem, you can learn problem-solving strategies and begin to develop some chemical intuition. Many of the problems you will solve in this course can be thought of as *unit conversion problems*, where you are given one or more quantities and asked to convert them into different units. Other problems require the use of *specific equations* to get to the information you are trying to find. In the sections that follow, we examine strategies to help you solve both of these types of problems. Of course, many problems contain both conversions and equations, requiring the combination of these strategies, and some problems may require an altogether different approach but the basic tools you learn here can be applied to those problems as well.

CONVERTING BETWEEN UNITS

Using units as a guide to solving problems is called dimensional analysis.

Units are critical in calculations. Knowing how to work with and manipulate units in calculations is a very important part of problem solving. In calculations, units help determine correctness. Units should always be included in calculations, and we can think of many calculations as converting from one unit to another. Units are multiplied, divided, and canceled like any other algebraic quantity.

Remember:

1. Always write every number with its associated unit. Never ignore units; they are critical.
2. Always include units in your calculations, dividing them and multiplying them as if they were algebraic quantities. Do not let units magically appear or disappear in calculations. Units must flow logically from beginning to end.

Consider converting 17.6 in. to centimeters. We know from Table 2.3 that 1 in. = 2.54 cm. To determine how many centimeters are in 17.6 in., we perform the conversion:

$$17.6\ \cancel{\text{in.}} \times \frac{2.54\,\text{cm}}{1\ \cancel{\text{in.}}} = 44.7\,\text{cm}$$

The unit *in.* cancels and we are left with *cm* as our final unit. The quantity $\frac{2.54\,\text{cm}}{1\,\text{in.}}$ is a **conversion factor** between *in.* and *cm*—it is a quotient with *cm* on top and *in.* on bottom.

For most conversion problems, we are given a quantity in some unit and asked to convert the quantity to another unit. These calculations take the form:

$$\text{information given} \times \text{conversion factor(s)} = \text{information sought}$$

$$\cancel{\text{given unit}} \times \frac{\text{desired unit}}{\cancel{\text{given unit}}} = \text{desired unit}$$

Conversion factors are constructed from any two quantities known to be equivalent. In our example, $2.54\,\text{cm} = 1\,\text{in.}$, so we construct the conversion factor by dividing both sides of the equality by 1 in. and canceling the units.

$$2.54\text{ cm} = 1\text{ in.}$$

$$\frac{2.54\text{ cm}}{1\text{ in.}} = \frac{1\ \cancel{\text{in.}}}{1\ \cancel{\text{in.}}}$$

$$\frac{2.54\text{ cm}}{1\text{ in.}} = 1$$

The quantity $\frac{2.54\text{ cm}}{1\text{ in.}}$ is equal to 1 and can be used to convert between inches and centimeters.

What if we want to perform the conversion the other way, from centimeters to inches? If we try to use the same conversion factor, the units do not cancel correctly.

$$44.7\,\text{cm} \times \frac{2.54\text{ cm}}{1\text{ in.}} = \frac{114\text{ cm}^2}{\text{in.}}$$

The units in the answer, as well as the value of the answer, are incorrect. The unit $\text{cm}^2/\text{in.}$ is not correct, and, based on our knowledge that centimeters are smaller than inches, we know that 44.7 cm cannot be equivalent to 114 in. In solving problems, always check if the final units are correct, and consider whether or not the magnitude of the answer makes sense. In this case, our mistake was in how we used the conversion factor. We must invert it.

$$44.7\ \cancel{\text{cm}} \times \frac{1\text{ in.}}{2.54\ \cancel{\text{cm}}} = 17.6\text{ in.}$$

Conversion factors can be inverted because they are equal to 1 and the inverse of 1 is 1.

$$\frac{1}{1} = 1$$

Therefore,

$$\frac{2.54\text{ cm}}{1\text{ in.}} = 1 = \frac{1\text{ in.}}{2.54\text{ cm}}$$

We can diagram conversions using a **solution map.** A solution map is a visual outline that shows the strategic route required to solve a problem. For unit conversion, the solution map focuses on units and how to convert from one unit to another. The solution map for converting from inches to centimeters is:

$$\frac{2.54\text{ cm}}{1\text{ in.}}$$

The solution map for converting from centimeters to inches is:

$$\frac{1\text{ in.}}{2.54\text{ cm}}$$

Each arrow in a solution map for a unit conversion has an associated conversion factor with the units of the previous step in the denominator and the units of the following step in the numerator. For one-step problems such as these, the solution map is only moderately helpful, but for multistep problems, it becomes a powerful way to develop a problem-solving strategy. In the section that follows, you will learn how to incorporate solution maps into an overall problem-solving strategy.

GENERAL PROBLEM-SOLVING STRATEGY

In this book, we use a standard problem-solving procedure that can be adapted to many of the problems encountered in chemistry and beyond. Solving any problem essentially requires you to assess the information given in the problem and devise a way to get to the information asked for. In other words, you need to

- Identify the starting point (the ***given*** information).
- Identify the end point (what you must ***find***).
- Devise a way to get from the starting point to the end point using what is given as well as what you already know or can look up. You can use a *solution map* to diagram the steps required to get from the starting point to the end point.

In graphic form, we can represent this progression as

Given $\longrightarrow$ **Solution Map** $\longrightarrow$ **Find**

One of the main difficulties beginning students have when trying to solve problems in general chemistry is not knowing where to start. Although no problem-solving procedure is applicable to all problems, the following four-step procedure can be helpful in working through many of the numerical problems you will encounter in this book.

1. **Sort.** Begin by sorting the information in the problem. *Given* information is the basic data provided by the problem—often one or more numbers with their associated units. The given information is the starting point for the problem. *Find* indicates what the problem is asking you to find (the end point of the problem).
2. **Strategize.** This is usually the hardest part of solving a problem. In this step, you must create a solution map—the series of steps that will get you from the given information to the information you are trying to find. You have already seen solution maps for simple unit conversion problems. Each arrow in a solution map represents a computational step. On the left side of the arrow is the quantity (or quantities) you had before the step; on the right side of the arrow is the quantity (or quantities) you will have after the step; and below the arrow is the information you need to get from one to the other—the relationship between the quantities.

   Often such relationships will take the form of conversion factors or equations. These may be given in the problem, in which case you will have written them down under "Given" in Step 1. Usually, however, you will need other information—which may include physical constants, formulas, or conversion factors—to help get you from what you are given to what you must find. You may recall this information from what you have learned or you can look it up in the chapters or tables within the book.

   In some cases, you may get stuck at the strategize step. If you cannot figure out how to get from the given information to the information you are asked to find, you might try working backwards. For example, you may want to look at the units of the quantity you are trying to find and look for conversion factors to get to the units of the given quantity. You may even try a combination of strategies; work forward, backward, or some of both. If you persist, you will develop a strategy to solve the problem.

3. **Solve.** This is the easiest part of solving a problem. Once you set up the problem properly and devise a solution map, you follow the map to solve the problem. Carry out mathematical operations (paying attention to the rules for significant figures in calculations) and cancel units as needed.
4. **Check.** This is the step most often overlooked by beginning students. Experienced problem solvers always ask, Does this answer make physical sense? Are the units correct? Is the number of significant figures correct? When solving multistep problems, errors easily creep into the solution. You can catch most of these errors by simply checking the answer. For example, suppose you are calculating the number of atoms in a gold coin and end up with an answer of 1.1×10^{-6} atoms. Could the gold coin really be composed of one-millionth of one atom?

In Examples 2.8 and 2.9, you will find this problem-solving procedure applied to unit conversion problems. The procedure is summarized in the left column, and two examples of applying the procedure are shown in the middle and right columns. This three-column format is used in selected examples throughout this text. It allows you to see how a particular procedure can be applied to two different problems. Work through one problem first (from top to bottom) and then examine how the same procedure is applied to the other problem. Recognizing the commonalities and differences between problems is a key part of problem solving.

PROBLEM-SOLVING PROCEDURE	**EXAMPLE 2.8** **Unit Conversion** Convert 7.8 km to miles.	**EXAMPLE 2.9** **Unit Conversion** Convert 0.825 m to millimeters.
SORT Begin by sorting the information in the problem into *given* and *find.*	**GIVEN:** 7.8 km **FIND:** mi	**GIVEN:** 0.825 m **FIND:** mm
STRATEGIZE Draw a *solution map* for the problem. Begin with the *given* quantity and symbolize each step with an arrow. Below the arrow, write the conversion factor for that step. The solution map ends at the *find* quantity. (In these examples, the relationships used in the conversions are below the solution map.)	**SOLUTION MAP** $\text{km} \xrightarrow[\frac{0.6214\ \text{mi}}{1\ \text{km}}]{} \text{mi}$ **RELATIONSHIPS USED** $1\,\text{km} = 0.6214\ \text{mi}$ (This conversion factor is from Table 2.3.)	**SOLUTION MAP** $\text{m} \xrightarrow[\frac{1\ \text{mm}}{10^{-3}\ \text{m}}]{} \text{mm}$ **RELATIONSHIPS USED** $1\,\text{mm} = 10^{-3}\ \text{m}$ (This conversion factor is from Table 2.2.)
SOLVE Follow the *solution map* to solve the problem. Begin with the *given* quantity and its units. Multiply by the appropriate conversion factor, canceling units to arrive at the *find* quantity.	**SOLUTION** $7.8\ \cancel{\text{km}} \times \frac{0.6214\ \text{mi}}{1\ \cancel{\text{km}}} = 4.84692\ \text{mi}$ $4.84692\ \text{mi} = 4.8\ \text{mi}$	**SOLUTION** $0.825\ \cancel{\text{m}} \times \frac{1\,\text{mm}}{10^{-3}\ \cancel{\text{m}}} = 825\ \text{mm}$ $825\ \text{mm} = 825\ \text{mm}$
Round the answer to the correct number of significant figures. (If possible, obtain conversion factors to enough significant figures so that they do not limit the number of significant figures in the answer.)	Round the answer to two significant figures, since the quantity given has two significant figures.	Leave the answer with three significant figures, since the quantity given has three significant figures and the conversion factor is a definition and therefore does not limit the number of significant figures in the answer.

CHECK Check your answer. Are the units correct? Does the answer make physical sense?	The units, mi, are correct. The magnitude of the answer is reasonable. A mile is longer than a kilometer, so the value in miles should be smaller than the value in kilometers.	The units, mm, are correct and the magnitude is reasonable. A millimeter is shorter than a meter, so the value in millimeters should be larger than the value in meters.
	▶**SKILLBUILDER 2.8** **Unit Conversion** Convert 56.0 cm to inches.	▶**SKILLBUILDER 2.9** **Unit Conversion** Convert 5678 m to kilometers.
	▶**FOR MORE PRACTICE** Example 2.25; Problems 73, 74, 75, 76.	▶**FOR MORE PRACTICE** Problems 69, 70, 71, 72.

CONCEPTUAL CHECKPOINT 2.5

Which conversion factor would you use to convert a distance in meters to kilometers?

(a) $\dfrac{1\ \text{m}}{10^3\ \text{km}}$

(b) $\dfrac{10^3\ \text{m}}{1\ \text{km}}$

(c) $\dfrac{1\ \text{km}}{10^3\ \text{m}}$

(d) $\dfrac{10^3\ \text{km}}{1\ \text{m}}$

2.7 Solving Multistep Unit Conversion Problems

When solving multistep unit conversion problems, we follow the preceding procedure, but we add more steps to the solution map. Each step in the solution map should have a conversion factor with the units of the previous step in the denominator and the units of the following step in the numerator. For example, suppose we want to convert 194 cm to feet. The solution map begins with cm, and we use the relationship 2.54 cm = 1 in to convert to in. We then use the relationship 12 in. = 1 ft to convert to ft.

SOLUTION MAP

$$\text{cm} \xrightarrow{\quad} \text{in.} \xrightarrow{\quad} \text{ft}$$

$$\frac{1\ \text{in.}}{2.54\ \text{cm}} \qquad\qquad \frac{1\ \text{ft}}{12\ \text{in.}}$$

Conversion factor   Conversion factor

Once the solution map is complete, we follow it to solve the problem.

SOLUTION

$$194\ \cancel{\text{cm}} \times \frac{1\ \cancel{\text{in.}}}{2.54\ \cancel{\text{cm}}} \times \frac{1\ \text{ft}}{12\ \cancel{\text{in.}}} = 6.3648\ \text{ft}$$

Since 1 foot is defined as 12 in., it does not limit significant figures.

We then round to the correct number of significant figures—in this case, three (from 194 cm, which has three significant figures).

$$6.3648 \text{ ft} = 6.36 \text{ ft}$$

Finally, we check the answer. The units of the answer, feet, are the correct ones, and the magnitude seems about right. Since a foot is larger than a centimeter, it is reasonable that the value in feet is smaller than the value in centimeters.

EXAMPLE 2.10 Solving Multistep Unit Conversion Problems

A recipe for making creamy pasta sauce calls for 0.75 L of cream. Your measuring cup measures only in cups. How many cups of cream should you use? (4 cups = 1 quart)

SORT
Begin by sorting the information in the problem into given and find.

GIVEN: 0.75 L
FIND: cups

STRATEGIZE
Draw a solution map for the problem. Begin with the *given* quantity and symbolize each step with an arrow. Below the arrow, write the conversion factor for that step. The solution map ends at the find quantity.

SOLUTION MAP

$$\text{L} \xrightarrow[\frac{1.057 \text{ qt}}{1 \text{ L}}]{} \text{qt} \xrightarrow[\frac{4 \text{ cups}}{1 \text{ qt}}]{} \text{cups}$$

RELATIONSHIPS USED
1.057 qt = 1 L (from Table 2.3.)
4 cups = 1 qt (given in problem statement)

SOLVE
Follow the solution map to solve the problem. Begin with 0.75 L and multiply by the appropriate conversion factor, canceling units to arrive at qt. Then, use the second conversion factor to arrive at cups.

Round the answer to the correct number of significant figures. In this case, round the answer to two significant figures, since the quantity given has two significant figures.

SOLUTION

$$0.75 \cancel{\text{L}} \times \frac{1.057 \cancel{\text{qt}}}{1 \cancel{\text{L}}} \times \frac{4 \text{ cups}}{1 \cancel{\text{qt}}} = 3.171 \text{ cups}$$

$$3.171 \text{ cups} = 3.2 \text{ cups}$$

CHECK
Check your answer. Are the units correct? Does the answer make physical sense?

The answer has the right units (cups) and seems reasonable. A cup is smaller than a liter, so the value in cups should be larger than the value in liters.

▶SKILLBUILDER 2.10 | Solving Multistep Unit Conversion Problems
A recipe calls for 1.2 cups of oil. How many liters of oil is this?

▶FOR MORE PRACTICE Problems 85, 86.

EXAMPLE 2.11 Solving Multistep Unit Conversion Problems

One lap of a running track measures 255 m. To run 10.0 km, how many laps should you run?

SORT
Begin by sorting the information in the problem into given and find. You are given a distance in km and asked to find the distance in laps. You are also given the quantity 255 m per lap, which is a conversion factor between m and laps.

GIVEN: 10.0 km
255 m = 1 lap
FIND: number of laps

STRATEGIZE
Build the solution map beginning with km and ending at laps. Focus on the units.

SOLUTION MAP

$$\text{km} \longrightarrow \text{m} \longrightarrow \text{laps}$$

$$\frac{10^3\ \text{m}}{1\ \text{km}} \qquad \frac{1\ \text{lap}}{255\ \text{m}}$$

RELATIONSHIPS USED

$1\,\text{km} = 10^3\,\text{m}$ (from Table 2.2)
$1\,\text{lap} = 255\,\text{m}$ (given in problem)

SOLVE
Follow the solution map to solve the problem. Begin with 10.0 km and multiply by the appropriate conversion factor, canceling units to arrive at m. Then, use the second conversion factor to arrive at laps. Round the intermediate answer (in blue) to three significant figures because it is limited by the three significant figures in the given quantity, 10.0 km.

SOLUTION

$$10.0\ \cancel{\text{km}} \times \frac{10^3\ \cancel{\text{m}}}{1\ \cancel{\text{km}}} \times \frac{1\,\text{lap}}{255\ \cancel{\text{m}}} = 39.216\ \text{laps} = 39.2\ \text{laps}$$

CHECK
Check your answer. Are the units correct? Does the answer make physical sense?

The units of the answer are correct, and the value of the answer makes sense: If a lap is 255 m, there are about 4 laps to each km (1000 m), so it seems reasonable that you would have to run about 40 laps to cover 10 km.

▶SKILLBUILDER 2.11 | Solving Multistep Unit Conversion Problems
A running track measures 1056 ft per lap. To run 15.0 km, how many laps should you run? ($1\,\text{mi} = 5280\,\text{ft}$)

▶SKILLBUILDER PLUS
An island is 5.72 nautical mi from the coast. How far is the island in meters? ($1\,\text{nautical mi} = 1.151\,\text{mi}$)

▶FOR MORE PRACTICE Problems 83, 84.

2.8 Units Raised to a Power

The unit cm^3 is often abbreviated as cc.

When converting quantities with units raised to a power, such as cubic centimeters (cm^3), *the conversion factor must also be raised to that power.* For example, suppose we want to convert the size of a motorcycle engine reported as $1255\ \text{cm}^3$ to cubic inches. We know that

$$2.54\,\text{cm} = 1\,\text{in.}$$

Most tables of conversion factors do not include conversions between cubic units, but we can derive them from the conversion factors for the basic units. We cube both sides of the preceding equality to obtain the proper conversion factor.

$2.54\,\text{cm} = 1\,\text{in.}$ is an exact conversion factor. After cubing, we retain five significant figures so that the conversion factor does not limit the four significant figures of our original quantity ($1255\ \text{cm}^3$).

$$(2.54\ \text{cm})^3 = (1\,\text{in.})^3$$
$$(2.54)^3\ \text{cm}^3 = 1^3\,\text{in.}^3$$
$$16.387\ \text{cm}^3 = 1\ \text{in.}^3$$

We can do the same thing in fractional form.

$$\frac{1\,\text{in.}}{2.54\ \text{cm}} = \frac{(1\,\text{in.})^3}{(2.54\ \text{cm})^3} = \frac{1\,\text{in.}^3}{16.387\ \text{cm}^3}$$

We then proceed with the conversion in the usual manner.

CHEMISTRY AND HEALTH

Drug Dosage

The unit of choice in specifying drug dosage is the milligram (mg). Pick up a bottle of aspirin, Tylenol, or any other common drug, and the label tells you the number of milligrams of the active ingredient contained in each tablet, as well as the number of tablets to take per dose. The following table shows the mass of the active ingredient per pill in several common pain relievers, all reported in milligrams. The remainder of each tablet is composed of inactive ingredients such as cellulose (or fiber) and starch.

The recommended adult dose for many of these pain relievers is one or two tablets every 4 to 8 hours (depending on the specific pain reliever). Notice that the extra-strength version of each pain reliever just contains a higher dose of the same compound found in the regular-strength version. For the pain relievers listed, three regular-strength tablets are the equivalent of two extra-strength tablets (and probably cost less).

The dosages given in the table are fairly standard for each drug, regardless of the brand. When you look on your drugstore shelf, you will find many different brands of regular-strength ibuprofen, some sold under the generic name and others sold under their brand names (such as Advil). However, if you look closely at the labels, you will find that they all contain the same thing: 200 mg of the compound ibuprofen. There is no difference in the compound or in the amount of the compound. Yet these pain relievers will most likely all have different prices. Choose the least expensive. Why pay more for the same thing?

CAN YOU ANSWER THIS? *Convert each of the doses in the table to ounces. Why are drug dosages not listed in ounces?*

Drug Mass per Pill for Common Pain Relievers

Pain Reliever	Mass of Active Ingredient per Pill
Aspirin	325 mg
Aspirin, extra strength	500 mg
Ibuprofen (Advil)	200 mg
Ibuprofen, extra strength	300 mg
Acetaminophen (Tylenol)	325 mg
Acetaminophen, extra strength	500 mg

SOLUTION MAP

$$\text{cm}^3 \xrightarrow{\qquad} \text{in.}^3$$

$$\frac{1\ \text{in.}^3}{16.387\ \text{cm}^3}$$

SOLUTION

$$1255\ \cancel{\text{cm}^3} \times \frac{1\ \text{in.}^3}{16.387\ \cancel{\text{cm}^3}} = 76.5851\ \text{in.}^3 = 76.59\ \text{in.}^3$$

EXAMPLE 2.12 Converting Quantities Involving Units Raised to a Power

A circle has an area of 2659 cm^2. What is its area in square meters?

SORT

You are given an area in square centimeters and asked to convert the area to square meters.

GIVEN: 2659 cm^2

FIND: m^2

STRATEGIZE

Build a solution map beginning with cm^2 and ending with m^2. Remember that you must square the conversion factor.

SOLUTION MAP

$$\text{cm}^2 \xrightarrow{\qquad} \text{m}^2$$

$$\frac{(0.01\ \text{m})^2}{(1\ \text{cm})^2}$$

RELATIONSHIPS USED

$1\ \text{cm} = 0.01\ \text{m}$ (from Table 2.2)

SOLVE

Follow the solution map to solve the problem. Square the conversion factor (both the units and the number) as you carry out the calculation.

Round the answer to four significant figures to reflect the four significant figures in the given quantity. The conversion factor is exact and therefore does not limit the number of significant figures.

SOLUTION

$$2659\,\text{cm}^2 \times \frac{(0.01\,\text{m})^2}{(1\,\text{cm})^2}$$

$$= 2659\,\cancel{\text{cm}^2} \times \frac{10^{-4}\,\text{m}^2}{1\,\cancel{\text{cm}^2}}$$

$$= 0.265900\,\text{m}^2$$

$$= 0.2659\,\text{m}^2$$

CHECK

Check your answer. Are the units correct? Does the answer make physical sense?

The units of the answer are correct, and the magnitude makes physical sense. A square meter is much larger than a square centimeter, so the value in square meters should be much smaller than the value in square centimeters.

▶SKILLBUILDER 2.12 | Converting Quantities Involving Units Raised to a Power

An automobile engine has a displacement (a measure of the size of the engine) of 289.7 in.^3 What is its displacement in cubic centimeters?

▶FOR MORE PRACTICE Example 2.26; Problems 87, 88, 89, 90, 91, 92.

EXAMPLE 2.13 Solving Multistep Conversion Problems Involving Units Raised to a Power

The average annual per person crude oil consumption in the United States is 15,615 dm^3. What is this value in cubic inches?

SORT

You are given a volume in cubic decimeters and asked to convert it to cubic inches.

GIVEN: 15,615 dm^3

FIND: in.^3

STRATEGIZE

Build a solution map beginning with dm^3 and ending with in.^3 Each of the conversion factors must be cubed, since the quantities involve cubic units.

SOLUTION MAP

$$\text{dm}^3 \xrightarrow{\frac{(0.1\text{ m})^3}{(1\text{ dm})^3}} \text{m}^3 \xrightarrow{\frac{(1\text{ cm})^3}{(0.01\text{ m})^3}} \text{cm}^3 \xrightarrow{\frac{(1\text{ in.})^3}{(2.54\text{ cm})^3}} \text{in.}^3$$

RELATIONSHIPS USED

1 dm = 0.1 m (from Table 2.2)
1 cm = 0.01 m (from Table 2.2)
2.54 cm = 1 in. (from Table 2.3)

SOLVE

Follow the solution map to solve the problem. Begin with the given value in dm^3 and multiply by the string of conversion factors to arrive at in^3. Make sure to cube each conversion factor as you carry out the calculation.

Round the answer to five significant figures to reflect the five significant figures in the least precisely known quantity (15,615 dm^3). The conversion factors are all exact and therefore do not limit the number of significant figures.

SOLUTION

$$15{,}615\,\cancel{\text{dm}}^3 \times \frac{(0.1\,\cancel{\text{m}})^3}{(1\,\cancel{\text{dm}})^3} \times \frac{(1\,\cancel{\text{cm}})^3}{(0.01\,\cancel{\text{m}})^3} \times \frac{(1\,\text{in.})^3}{(2.54\,\cancel{\text{cm}})^3}$$

$$= 9.5289 \times 10^5\,\text{in.}^3$$

CHECK

Check your answer. Are the units correct? Does the answer make physical sense?

The units of the answer are correct and the magnitude makes sense. A cubic inch is smaller than a cubic decimeter, so the value in cubic inches should be larger than the value in cubic decimeters.

▶SKILLBUILDER 2.13 | Solving Multistep Problems Involving Units Raised to a Power

How many cubic inches are there in 3.25 yd^3?

▶FOR MORE PRACTICE Problems 93, 94.

CONCEPTUAL CHECKPOINT 2.6

You know that there are 3 ft in a yard. How many cubic feet are there in a cubic yard?

(a) 3

(b) 6

(c) 9

(d) 27

2.9 Density

▲ Top-end bicycle frames are made of titanium because of titanium's low density and high relative strength. Titanium has a density of 4.50 g/cm^3, while iron, for example, has a density of 7.86 g/cm^3.

Why do some people pay more than $3000 for a bicycle made of titanium? A steel frame would be just as strong for a fraction of the cost. The difference between the two bikes is their mass—the titanium bike is lighter. For a given volume of metal, titanium has less mass than steel. We describe this property by saying that titanium is *less dense* than steel. The **density** of a substance is the ratio of its mass to its volume.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \text{or} \quad d = \frac{m}{V}$$

Density is a fundamental property of substances that differs from one substance to another. The units of density are those of mass divided by those of volume, most conveniently expressed in grams per cubic centimeter (g/cm^3) or grams per milliliter (g/mL). See Table 2.4 for a list of the densities of some common substances. Aluminum is among the least dense structural metals with a density of 2.70 g/cm^3, while platinum is among the densest with a density of 21.4 g/cm^3. Titanium has a density of 4.50 g/cm^3.

TABLE 2.4 Densities of Some Common Substances

Substance	Density (g/cm^3)
Charcoal, oak	0.57
Ethanol	0.789
Ice	0.92
Water	1.0
Glass	2.6
Aluminum	2.7
Titanium	4.50
Iron	7.86
Copper	8.96
Lead	11.4
Gold	19.3
Platinum	21.4

Remember that cubic centimeters and milliliters are equivalent units.

CALCULATING DENSITY

We calculate the density of a substance by dividing the mass of a given amount of the substance by its volume. For example, a sample of liquid has a volume of 22.5 mL and a mass of 27.2 g. To find its density, we use the equation $d = m/V$.

$$d = \frac{m}{V} = \frac{27.2 \text{ g}}{22.5 \text{ mL}} = 1.21 \text{ g/mL}$$

We can use a solution map for solving problems involving equations, but the solution map will take a slightly different form than for pure conversion problems. In a problem involving an equation, the solution map shows how the *equation* takes you from the *given* quantities to the *find* quantity. The solution map for this problem is:

$$d = \frac{m}{V}$$

The solution map illustrates how the values of m and V, when substituted into the equation $d = \frac{m}{V}$ give the desired result, d.

EXAMPLE 2.14 Calculating Density

A jeweler offers to sell a ring to a woman and tells her that it is made of platinum. Noting that the ring felt a little light, the woman decides to perform a test to determine the ring's density. She places the ring on a balance and finds that it has a mass of 5.84 g. She also finds that the ring *displaces* 0.556 cm^3 of water. Is the ring made of platinum? The density of platinum is 21.4 g/cm^3. (The displacement of water is a common way to measure the volume of irregularly shaped objects. To say that an object *displaces* 0.556 cm^3 of water means that when the object is submerged in a container of water filled to the brim, 0.556 cm^3 of water overflows. Therefore, the volume of the object is 0.556 cm^3.)

SORT
You are given the mass and volume of the ring and asked to find the density.

GIVEN: $m = 5.84$ g

$V = 0.556$ cm^3

FIND: density in g/cm^3

STRATEGIZE
If the ring is platinum, its density should match that of platinum. Build a solution map that represents how you get from the given quantities (mass and volume) to the find quantity (density). Unlike in conversion problems, where you write a conversion factor beneath the arrow, here you write the equation for density beneath the arrow.

SOLUTION MAP

$$\boxed{m, V} \xrightarrow[d = \frac{m}{V}]{} \boxed{d}$$

RELATIONSHIPS USED

$$d = \frac{m}{V} \quad \text{(equation for density)}$$

SOLVE
Follow the solution map. Substitute the given values into the density equation and compute the density.

Round the answer to three significant figures to reflect the three significant figures in the given quantities.

SOLUTION

$$d = \frac{m}{V} = \frac{5.84\text{ g}}{0.556\text{ cm}^3} = 10.5\text{ g/cm}^3$$

The density of the ring is much too low to be platinum; therefore the ring is a fake.

CHECK
Check your answer. Are the units correct? Does the answer make physical sense?

The units of the answer are correct, and the magnitude seems reasonable to be an actual density. As you can see from Table 2.4, the densities of liquids and solids range from below 1 g/cm^3 to just over 20 g/cm^3.

►SKILLBUILDER 2.14 | Calculating Density

The woman takes the ring back to the jewelry shop, where she is met with endless apologies. They accidentally had made the ring out of silver rather than platinum. They give her a new ring that they promise is platinum. This time when she checks the density, she finds the mass of the ring to be 9.67 g and its volume to be 0.452 cm^3. Is this ring genuine?

►FOR MORE PRACTICE Example 2.27; Problems 95, 96, 97, 98, 99, 100.

DENSITY AS A CONVERSION FACTOR

We can use the density of a substance as a conversion factor between the mass of the substance and its volume. For example, suppose we need 68.4 g of a liquid with a density of 1.32 g/cm^3 and want to measure the correct amount with a graduated cylinder (a piece of laboratory glassware used to measure volume). How much volume should we measure?

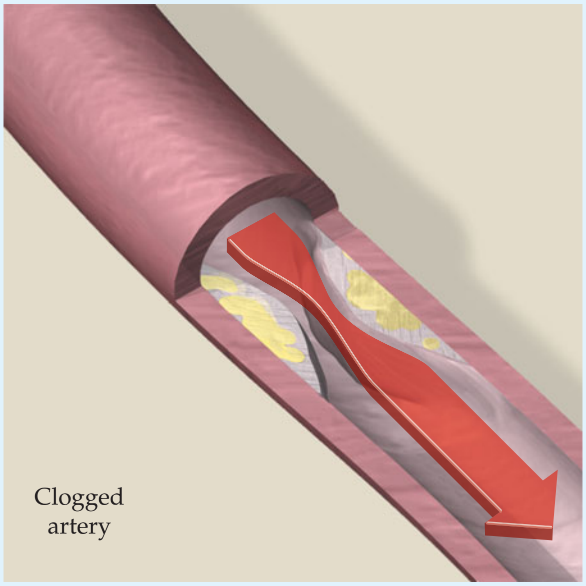
We start with the mass of the liquid and use the density as a conversion factor to convert mass to volume. However, we must use the inverted density expression 1 cm^3/1.32 g because we want g, the unit we are converting from, to be on the bottom (in the denominator) and cm^3, the unit we are converting to, on the top (in the numerator). Our solution map takes this form:

CHEMISTRY AND HEALTH

Density, Cholesterol, and Heart Disease

Cholesterol is fatty substance found in animal-derived foods such as beef, eggs, fish, poultry, and milk products. Cholesterol is used by the body for several purposes. However, excessive amounts in the blood—which can be caused by both genetic factors and diet—may result in the deposition of cholesterol in arterial walls, leading to a condition called atherosclerosis, or blocking of the arteries. These blockages are dangerous because they inhibit blood flow to important organs, causing heart attacks and strokes. The risk of stroke and heart attack increases with increasing blood cholesterol levels (Table 2.5). Cholesterol is carried in the bloodstream by a class of substances known as lipoproteins. Lipoproteins are often separated and classified according to their density.

The main carriers of blood cholesterol are low-density lipoproteins (LDLs). LDLs, also called bad cholesterol, have a density of 1.04 g/cm^3. They are bad because they tend to deposit cholesterol on arterial walls, increasing the risk of stroke and heart attack. Cholesterol is also carried by high-density lipoproteins (HDLs). HDLs, also called good cholesterol, have a density of 1.13 g/cm^3. HDLs transport cholesterol to the liver for processing and excretion and therefore have a tendency to reduce cholesterol on arterial walls. Too low a level of HDLs (below 35 mg/100 mL) is considered a risk factor for heart disease. Exercise, along with a diet low in saturated fats, is believed to raise HDL levels in the blood while lowering LDL levels.



▲ Too many low-density lipoproteins in the blood can lead to the blocking of arteries.

CAN YOU ANSWER THIS? *What mass of low-density lipoprotein is contained in a cylinder that is 1.25 cm long and 0.50 cm in diameter? (The volume of a cylinder, V, is given by $V = \pi r^2 \ell$, where r is the radius of the cylinder and ℓ is its length.)*

TABLE 2.5 Risk of Stroke and Heart Attack vs. Blood Cholesterol Level

Risk Level	Total Blood Cholesterol (mg/100 mL)	LDL (mg/100 mL)
low	<200	<130
borderline	200–239	130–159
high	240+	160+

SOLUTION MAP

$$\text{g} \xrightarrow{\frac{1\ \text{cm}^3}{1.32\ \text{g}}} \text{cm}^3 \xrightarrow{\frac{1\ \text{mL}}{1\ \text{cm}^3}} \text{mL}$$

SOLUTION

$$68.4\ \cancel{\text{g}} \times \frac{1\ \cancel{\text{cm}^3}}{1.32\ \cancel{\text{g}}} \times \frac{1\ \text{mL}}{1\ \cancel{\text{cm}^3}} = 51.8\ \text{mL}$$

We must measure 51.8 mL to obtain 68.4 g of the liquid.

EXAMPLE 2.15 Density as a Conversion Factor

The gasoline in an automobile gas tank has a mass of 60.0 kg and a density of $0.752\ \text{g/cm}^3$. What is its volume in cm^3?

SORT

You are given the mass in kilograms and asked to find the volume in cubic centimeters. Density is the conversion factor between mass and volume.

GIVEN: 60.0 kg

$$\text{Density} = 0.752\ \text{g/cm}^3$$

FIND: volume in cm^3

STRATEGIZE

Build the solution map starting with kg and ending with cm^3. Use the density (inverted) to convert from g to cm^3.

SOLUTION MAP

$$\text{kg} \xrightarrow[\frac{1000\ \text{g}}{1\ \text{kg}}]{} \text{g} \xrightarrow[\frac{1\ \text{cm}^3}{0.752\ \text{g}}]{} \text{cm}^3$$

RELATIONSHIPS USED

$0.752\ \text{g/cm}^3$ (given in problem)

$1000\ \text{g} = 1\ \text{kg}$ (from Table 2.2)

SOLVE

Follow the solution map to solve the problem. Round the answer to three significant figures to reflect the three significant figures in the given quantities.

SOLUTION

$$60.0\ \cancel{\text{kg}} \times \frac{1000\ \cancel{\text{kg}}}{1\ \cancel{\text{kg}}} \times \frac{1\ \text{cm}^3}{0.752\ \cancel{\text{g}}} = 7.98 \times 10^4\ \text{cm}^3$$

CHECK

Check your answer. Are the units correct? Does the answer make physical sense?

The units of the answer are those of volume, so they are correct. The magnitude seems reasonable because the density is somewhat less than $1\ \text{g/cm}^3$; therefore the volume of 60.0 kg should be somewhat more than $60.0 \times 10^3\ \text{cm}^3$.

▶SKILLBUILDER 2.15 | Density as a Conversion Factor

A drop of acetone (nail polish remover) has a mass of 35 mg and a density of $0.788\ \text{g/cm}^3$. What is its volume in cubic centimeters?

▶SKILLBUILDER PLUS

A steel cylinder has a volume of $246\ \text{cm}^3$ and a density of $7.93\ \text{g/cm}^3$. What is its mass in kilograms?

▶FOR MORE PRACTICE Example 2.28; Problems 101, 102.

2.10 Numerical Problem-Solving Overview

In this chapter, you have seen a few examples of how to solve numerical problems. In Section 2.6, we developed a procedure to solve simple unit conversion problems. We then learned how to modify that procedure to work with multistep unit conversion problems and problems involving an equation. We will now summarize and generalize these procedures and apply them to two additional examples. As we did in Section 2.6, we provide the general procedure for solving numerical problems in the left column and the application of the procedure to two examples in the center and right columns.

SOLVING NUMERICAL PROBLEMS

SORT

- Scan the problem for one or more numbers and their associated units. This number (or numbers) is (are) the starting point(s) of the calculation. Write them down as given.
- Scan the problem to determine what you are asked to find. Sometimes the units of this quantity are implied; other times they are specified. Write down the quantity and/or units you are asked to find.

STRATEGIZE

- For problems involving only conversions, focus on units. The solution map shows how to get from the units in the given quantity to the units in the quantity you are asked to find.
- For problems involving equations, focus on the equation. The solution map shows how the equation takes you from the given quantity (or quantities) to the quantity you are asked to find.
- Some problems may involve both unit conversions and equations, in which case the solution map employs both of the above points.

SOLVE

- For problems involving only conversions, begin with the given quantity and its units. Multiply by the appropriate conversion factor(s), canceling units, to arrive at the quantity you are asked to find.

EXAMPLE 2.16

Unit Conversion

A 23.5-kg sample of ethanol is needed for a large-scale reaction. What volume in liters of ethanol should be used? The density of ethanol is 0.789 g/cm^3.

GIVEN: 23.5 kg ethanol

$$\text{density} = 0.789\,\text{g/cm}^3$$

FIND: volume in L

SOLUTION MAP

$$\text{kg} \xrightarrow{\frac{1000\text{ g}}{1\text{ kg}}} \text{g} \xrightarrow{\frac{1\text{ cm}^3}{0.789\text{ g}}} \text{cm}^3 \xrightarrow{\frac{1\text{ mL}}{1\text{ cm}^3}} \text{mL} \xrightarrow{\frac{1\text{L}}{1000\text{ mL}}} \text{L}$$

RELATIONSHIPS USED

$$0.789\,\text{g/cm}^3 \text{ (given in problem)}$$
$$1000\text{ g} = 1\,\text{kg (Table 2.2)}$$
$$1000\text{ mL} = 1\,\text{L (Table 2.2)}$$
$$1\text{ mL} = 1\text{ cm}^3 \text{ (Table 2.3)}$$

SOLUTION

$$23.5\ \cancel{\text{kg}} \times \frac{1000\ \cancel{\text{g}}}{1\ \cancel{\text{kg}}} \times \frac{1\ \cancel{\text{cm}^3}}{0.789\ \cancel{\text{g}}} \times \frac{1\ \cancel{\text{ml}}}{1\ \cancel{\text{cm}^3}} \times \frac{1\text{ L}}{1000\ \cancel{\text{mL}}} = 29.7845\text{ L}$$

$$29.7845\text{ L} = 29.8\text{ L}$$

EXAMPLE 2.17

Unit Conversion with Equation

A 55.9-kg person displaces 57.2 L of water when submerged in a water tank. What is the density of the person in grams per cubic centimeter?

GIVEN: $m = 55.9\,\text{kg}$

$V = 57.2\,\text{L}$

FIND: density in g/cm^3

SOLUTION MAP

$$m, V \xrightarrow{d = \frac{m}{V}} d$$

RELATIONSHIPS USED

$$d = \frac{m}{V} \text{ (definition of density)}$$

The equation is already solved for the find quantity. Convert mass from kilograms to grams.

$$m = 55.9\ \cancel{\text{kg}} \times \frac{1000\text{ g}}{1\ \cancel{\text{kg}}}$$
$$= 5.59 \times 10^4\,\text{g}$$

- For problems involving equations, solve the equation to arrive at the quantity you are asked to find. (Use algebra to rearrange the equation so that the quantity you are asked to find is isolated on one side.) Gather each of the quantities that must go into the equation in the correct units. (Convert to the correct units using additional solution maps if necessary.) Finally, substitute the numerical values and their units into the equation and compute the answer.
- Round the answer to the correct number of significant figures. Use the significant-figure rules from Sections 2.3 and 2.4.

Convert volume from liters to cubic centimeters.

$$V = 57.2\ \cancel{\text{L}} \times \frac{1000\ \cancel{\text{mL}}}{1\ \cancel{\text{L}}} \times \frac{1\ \text{cm}^3}{1\ \cancel{\text{mL}}}$$

$$= 57.2 \times 10^3\ \text{cm}^3$$

Compute density.

$$d = \frac{m}{V} = \frac{55.9 \times 10^3}{57.2 \times 10^3\ \text{cm}^3}$$

$$= 0.9772727\ \frac{\text{g}}{\text{cm}^3}$$

$$= 0.977\ \frac{\text{g}}{\text{cm}^3}$$

CHECK

- Does the magnitude of the answer make physical sense? Are the units correct?

The units are correct (L) and the magnitude is reasonable. Since the density is less than 1 g/cm^3, the computed volume (29.8 L) should be greater than the mass (23.5 kg).

The units are correct. Since the mass in kilograms and the volume in liters were very close to each other in magnitude, it makes sense that the density is close to 1 g/cm^3.

▶SKILLBUILDER 2.16

Unit Conversion

A pure gold metal bar displaces 0.82 L of water. What is its mass in kilograms? (The density of gold is 19.3 g/cm^3.)

▶FOR MORE PRACTICE Problems 103, 109, 110, 111, 112.

▶SKILLBUILDER 2.17

Unit Conversion with Equation

A gold-colored pebble is found in a stream. Its mass is 23.2 mg, and its volume is 1.20 mm^3. What is its density in grams per cubic centimeter? Is it gold? (The density of gold = 19.3 g/cm^3.)

▶FOR MORE PRACTICE Problems 104, 105, 106.

CHAPTER IN REVIEW

CHEMICAL PRINCIPLES

Uncertainty: Scientists report measured quantities so that the number of digits reflects the certainty in the measurement. Write measured quantities so that every digit is certain except the last, which is estimated.

RELEVANCE

Uncertainty: Measurement is a hallmark of science, and the precision of a measurement must be communicated with the measurement so that others know how reliable the measurement is. When you write or manipulate measured quantities, you must show and retain the precision with which the original measurement was made.

Units: Measured quantities usually have units associated with them. The SI unit for length is the meter; for mass, the kilogram; and for time, the second. Prefix multipliers such as *kilo-* or *milli-* are often used in combination with these basic units. The SI units of volume are units of length raised to the third power; liters or milliliters are often used as well.

Units: The units in a measured quantity communicate what the quantity actually is. Without an agreed-on system of units, scientists could not communicate their measurements. Units are also important in calculations, and the tracking of units throughout a calculation is essential.

Density: The density of a substance is its mass divided by its volume, $d = m/V$, and is usually reported in units of grams per cubic centimeter or grams per milliliter. Density is a fundamental property of all substances and generally differs from one substance to another.

Density: The density of substances is an important consideration in choosing materials from which to make things. Airplanes, for example, are made of low-density materials, while bridges are made of higher-density materials. Density is important as a conversion factor between mass and volume and vice versa.

CHEMICAL SKILLS

Scientific Notation (Section 2.2)

To express a number in scientific notation:

- Move the decimal point to obtain a number between 1 and 10.
- Write the decimal part multiplied by 10 raised to the number of places you moved the decimal point.
- The exponent is positive if you moved the decimal point to the left and negative if you moved the decimal point to the right.

EXAMPLES

EXAMPLE 2.18 Scientific Notation

Express the number 45,000,000 in scientific notation.

45,000,000

7 6 5 4 3 2 1

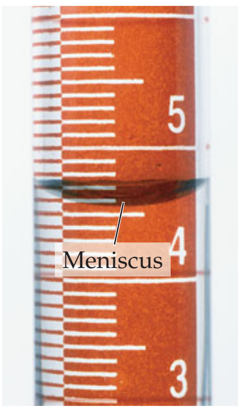
4.5×10^7

Reporting Measured Quantities to the Right Number of Digits (Section 2.3)

Report measured quantities so that every digit is certain except the last, which is estimated.

EXAMPLE 2.19 Reporting Measured Quantities to the Right Number of Digits

Record the volume of liquid in the graduated cylinder to the correct number of digits. Laboratory glassware is calibrated (and should therefore be read) from the bottom of the meniscus (see figure).



Since the graduated cylinder has markings every 0.1 mL, the measurement should be recorded to the nearest 0.01 mL. In this case, that is 4.57 mL.

Counting Significant Digits (Section 2.3)

The following digits should always be counted as significant:

- nonzero digits
- interior zeros
- trailing zeros after a decimal point
- trailing zeros before a decimal point but after a nonzero number

The following digits should never be counted as significant:

- zeros to the left of the first nonzero number

The following digits are ambiguous and should be avoided by using scientific notation:

- zeros at the end of a number, but before a decimal point

EXAMPLE 2.20 Counting Significant Digits

How many significant figures are in the following numbers?

1.0050	five significant figures
0.00870	three significant figures
100.085	six significant digits
5400	It is not possible to tell in its current form.

In order for us to know, the number needs to be written as 5.4×10^3, 5.40×10^3, or 5.400×10^3, depending on the number of significant figures intended.

Rounding (Section 2.4)

When rounding numbers to the correct number of significant figures, round down if the last digit dropped is 4 or less; round up if the last digit dropped is 5 or more.

EXAMPLE 2.21 Rounding

Round 6.442 and 6.456 to two significant figures each.

6.442 rounds to 6.4
6.456 rounds to 6.5

Significant Figures in Multiplication and Division (Section 2.4)

The result of a multiplication or division should carry the same number of significant figures as the factor with the least number of significant figures.

EXAMPLE 2.22 Significant Figures in Multiplication and Division

Perform the following calculation and report the answer to the correct number of significant figures.

$$\begin{aligned} &8.54 \times 3.589 \div 4.2 \\ &= 7.2976 \\ &= 7.3 \end{aligned}$$

Round the final result to two significant figures to reflect the two significant figures in the factor with the least number of significant figures (4.2).

Significant Figures in Addition and Subtraction (Section 2.4)

The result of an addition or subtraction should carry the same number of decimal places as the quantity carrying the least number of decimal places.

EXAMPLE 2.23 Significant Figures in Addition and Subtraction

Perform the following operation and report the answer to the correct number of significant figures.

$$\begin{array}{r} 3.098 \\ 0.67 \\ -0.9452 \\ \hline 2.8228 \end{array} = 2.82$$

Round the final result to two decimal places to reflect the two decimal places in the quantity with the least number of decimal places (0.67).

Significant Figures in Calculations Involving Both Addition/Subtraction and Multiplication/Division (Section 2.4)

In calculations involving both addition/subtraction and multiplication/division, do the steps in parentheses first, keeping track of how many significant figures are in the answer by underlining the least significant figure, then proceeding with the remaining steps. Do not round off until the very end.

EXAMPLE 2.24 Significant Figures in Calculations Involving Both Addition/Subtraction and Multiplication/Division

Perform the following operation and report the answer to the correct number of significant figures.

$$\begin{aligned} &8.16 \times (5.4323 - 5.411) \\ &= 8.16 \times 0.02\underline{1}3 \\ &= 0.1738 = 0.17 \end{aligned}$$

Unit Conversion (Sections 2.6, 2.7)

Solve unit conversion problems by following these steps.

1. **Sort** Write down the given quantity and its units and the quantity you are asked to find and its units.
2. **Strategize** Draw a solution map showing how to get from the given quantity to the quantity you are asked to find.
3. **Solve** Follow the solution map. Starting with the given quantity and its units, multiply by the appropriate conversion factor(s), canceling units, to arrive at the quantity to find in the desired units. Round the final answer to the correct number of significant figures.
4. **Check** Are the units correct? Does the answer make physical sense?

EXAMPLE 2.25 Unit Conversion

Convert 108 ft to meters.

GIVEN: 108 ft

FIND: m

SOLUTION MAP

$$\text{ft} \xrightarrow{\frac{12\ \text{in.}}{1\ \text{ft}}} \text{in.} \xrightarrow{\frac{1\ \text{m}}{39.37\ \text{in.}}} \text{m}$$

RELATIONSHIPS USED

1 m = 39.37 in. (Table 2.3)

1 ft = 12 in. (by definition)

SOLUTION

$$\begin{aligned} &108\ \cancel{\text{ft}} \times \frac{12\ \cancel{\text{in.}}}{1\ \cancel{\text{ft}}} \times \frac{1\ \text{m}}{39.37\ \cancel{\text{in.}}} \\ &= 32.918\ \text{m} \\ &= 32.9\ \text{m} \end{aligned}$$

The answer has the right units (meters), and it makes sense; since a meter is longer than a foot, the number of meters should be less than the number of feet.

Unit Conversion Involving Units Raised to a Power (Section 2.8)

When working problems involving units raised to a power, raise the conversion factors to the same power.

1. **Sort** Write down the given quantity and its units and the quantity you are asked to find and its units.
2. **Strategize** Draw a solution map showing how to get from the given quantity to the quantity you are asked to find. Since the units are squared, you must square the conversion factor.

EXAMPLE 2.26 Unit Conversion Involving Units Raised to a Power

How many square meters are in 1.0 km^2?

GIVEN: 1.0 km^2

FIND: m^2

SOLUTION MAP

$$\text{km}^2 \xrightarrow{\frac{(1000\ \text{m})^2}{(1\ \text{km})^2}} \text{m}^2$$

RELATIONSHIPS USED

1 km = 1000 m (Table 2.2)

3. **Solve** Follow the solution map. Starting with the given quantity and its units, multiply by the appropriate conversion factor(s), canceling units, to arrive at the quantity you are asked to find in the desired units. Don't forget to square the conversion factor for squared units.
4. **Check** Are the units correct? Does the answer make physical sense?

SOLUTION

$$1.0\,\text{km}^2 \times \frac{(1000\,\text{m})^2}{(1\,\text{km})^2}$$

$$= 1.0\,\cancel{\text{km}^2} \times \frac{1 \times 10^6\,\text{m}^2}{1\,\cancel{\text{km}^2}}$$

$$= 1.0 \times 10^6\,\text{m}^2$$

The units are correct. The answer makes physical sense; a square meter is much smaller than a square kilometer, so the number of square meters should be much larger than the number of square kilometers.

Calculating Density (Section 2.10)

The density of an object or substance is its mass divided by its volume.

$$d = \frac{m}{V}$$

EXAMPLE 2.27 Calculating Density

An object has a mass of 23.4 g and displaces 5.7 mL of water. Determine its density in grams per milliliter.

1. **Sort** Write down the given quantity and its units and the quantity you are asked to find and its units.

GIVEN:

$$m = 23.4\ \text{g}$$
$$V = 5.7\ \text{mL}$$

FIND: density in g/mL

2. **Strategize** Draw a solution map showing how to get from the given quantity to the quantity you are asked to find. Use the definition of density as the equation that takes you from the mass and the volume to the density.

SOLUTION MAP

$$m, V \longrightarrow d$$

$$d = \frac{m}{V}$$

RELATIONSHIPS USED

$$d = \frac{m}{V} \text{ (definition of density)}$$

3. **Solve** Substitute the correct values into the equation for density.

SOLUTION

$$d = \frac{m}{V}$$
$$= \frac{23.4\,\text{g}}{5.7\,\text{mL}}$$
$$= 4.11\,\text{g/mL}$$
$$= 4.1\ \text{g/mL}$$

4. **Check** Are the units correct? Does the answer make physical sense?

The units (g/mL) are units of density. The answer is in the range of values for the densities of liquids and solids (see Table 2.4).

Density as a Conversion Factor (Section 2.10)

Density can be used as a conversion factor from mass to volume or from volume to mass. To convert between volume and mass, use density directly. To convert between mass and volume, invert the density.

1. **Sort** Write down the given quantity and its units and the quantity you are asked to find and its units.
2. **Strategize** Draw a solution map showing how to get from the given quantity to the quantity you are asked to find. Use the inverse of the density to convert from g to mL.
3. **Solve** Begin with given quantity and multiply by the appropriate conversion factors to arrive at the quantity you are asked to find. Round to the correct number of significant figures.
4. **Check** Are the units correct? Does the answer make physical sense?

EXAMPLE 2.28 Density as a Conversion Factor

What is the volume in liters of 321 g of a liquid with a density of 0.84 g/mL?

GIVEN: 321 g

FIND: volume in L

SOLUTION MAP

$$\text{g} \xrightarrow{\frac{1\ \text{mL}}{0.84\ \text{g}}} \text{mL} \xrightarrow{\frac{1\ \text{L}}{1000\ \text{mL}}} \text{L}$$

RELATIONSHIPS USED

0.84 g/mL (given in the problem)
1 L = 1000 mL (Table 2.2)

SOLUTION

$$321\ \cancel{\text{g}} \times \frac{1\ \cancel{\text{mL}}}{0.84\ \cancel{\text{g}}} \times \frac{1\ \text{L}}{1000\ \cancel{\text{mL}}} = 0.382\ \text{L} = 0.38\ \text{L}$$

The answer is in the correct units. The magnitude seems right because the density is slightly less than 1; therefore the volume (382 mL) should be slightly greater than the mass (321 g).

KEY TERMS

conversion factor **[2.6]**
decimal part **[2.2]**
density **[2.9]**
English system **[2.5]**
exponent **[2.2]**
exponential part **[2.2]**
International System **[2.5]**
kilogram (kg) **[2.5]**
liter (L) **[2.5]**
mass **[2.5]**
meter (m) **[2.5]**
metric system **[2.5]**
prefix multipliers **[2.5]**
scientific notation **[2.2]**
second (s) **[2.5]**
SI units **[2.5]**
significant figures (digits) **[2.3]**
solution map **[2.6]**
units **[2.5]**
volume **[2.5]**

EXERCISES

QUESTIONS

Answers to all questions numbered in blue appear in the Answers section at the back of the book.

1. Why is it important to report units with scientific measurements?
2. Why are the number of digits reported in scientific measurements important?
3. Why is scientific notation useful?
4. If a measured quantity is written correctly, which digits are certain? Which are uncertain?
5. Explain when zeros count as significant digits and when they do not.
6. How many significant digits are there in exact numbers? What kinds of numbers are exact?
7. What limits the number of significant digits in a calculation involving only multiplication and division?
8. What limits the number of significant digits in a calculation involving only addition and subtraction?
9. How are significant figures determined in calculations involving both addition/subtraction and multiplication/division?
10. What are the rules for rounding numbers?
11. What are the basic SI units of length, mass, and time?
12. List the common units of volume.
13. Suppose you are trying to measure the diameter of a Frisbee. What unit and prefix multiplier should you use?
14. What is the difference between mass and weight?

15. Obtain a metric ruler and measure these objects to the correct number of significant figures.
   (a) quarter (diameter)
   (b) dime (diameter)
   (c) notebook paper (width)
   (d) this book (width)
16. Obtain a stopwatch and measure each time to the correct number of significant figures.
   (a) time between your heartbeats
   (b) time it takes you to do the next problem
   (c) time between your breaths
17. Explain why units are important in calculations.
18. How are units treated in a calculation?
19. What is a conversion factor?
20. Why is the fundamental value of a quantity not changed when the quantity is multiplied by a conversion factor?
21. Write the conversion factor that converts a measurement in inches to feet. How would the conversion factor change for converting a measurement in feet to inches?
22. Write conversion factors for each:
   (a) miles to kilometers
   (b) kilometers to miles
   (c) gallons to liters
   (d) liters to gallons
23. This book outlines a four-step problem-solving strategy. Describe each step and its significance.
   (a) Sort
   (b) Strategize
   (c) Solve
   (d) Check
24. Experienced problem solvers always consider both the value and units of their answer to a problem. Why?
25. Draw a solution map to convert a measurement in grams to pounds.
26. Draw a solution map to convert a measurement in milliliters to gallons.
27. Draw a solution map to convert a measurement in meters to feet.
28. Draw a solution map to convert a measurement in ounces to grams. (1 lb = 16 oz)
29. What is density? Explain why density can work as a conversion factor. Between what quantities does it convert?
30. Explain how you would calculate the density of a substance. Include a solution map in your explanation.

PROBLEMS

Note: The exercises in the Problems section are paired, and the answers to the odd-numbered exercises (numbered in blue) appear in the Answers section at the back of the book.

SCIENTIFIC NOTATION

31. Express each number in scientific notation.
   (a) 36,756,000 (population of California)
   (b) 1,288,000 (population of Hawaii)
   (c) 19,490,000 (population of New York)
   (d) 532,000 (population of Wyoming)
32. Express each number in scientific notation.
   (a) 6,796,000,000 (population of the world)
   (b) 1,338,000,000 (population of China)
   (c) 11,451,000 (population of Cuba)
   (d) 4,203,000 (population of Ireland)
33. Express each number in scientific notation.
   (a) 0.00000000007461 m (length of a hydrogen–hydrogen chemical bond)
   (b) 0.0000158 mi (number of miles in an inch)
   (c) 0.000000632 m (wavelength of red light)
   (d) 0.000015 m (diameter of a human hair)
34. Express each number in scientific notation.
   (a) 0.000000001 s (time it takes light to travel 1 ft)
   (b) 0.143 s (time it takes light to travel around the world)
   (c) 0.000000000001 s (time it takes a chemical bond to undergo one vibration)
   (d) 0.000001 m (approximate size of a dust particle)
35. Express each number in decimal notation (i.e., express the number without using scientific notation).
   (a) 6.022×10^{23} (number of carbon atoms in 12.01 g of carbon)
   (b) 1.6×10^{-19} C (charge of a proton in coulombs)
   (c) 2.99×10^{8} m/s (speed of light)
   (d) 3.44×10^{2} m/s (speed of sound)
36. Express each number in decimal notation (i.e., express the number without using scientific notation).
   (a) 450×10^{-19} m (wavelength of blue light)
   (b) 13.7×10^{9} years (approximate age of the universe)
   (c) 5×10^{9} years (approximate age of Earth)
   (d) 4.7×10^{1} years (approximate age of this author)
37. Express each number in decimal notation (i.e., express the number without using scientific notation).
   (a) 3.22×10^{7}
   (b) 7.2×10^{-3}
   (c) 1.18×10^{11}
   (d) 9.43×10^{-6}
38. Express each number in decimal notation. (i.e., express the number without using scientific notation)
   (a) 1.30×10^{6}
   (b) 1.1×10^{-4}
   (c) 1.9×10^{2}
   (d) 7.41×10^{-10}

39. Complete the table.

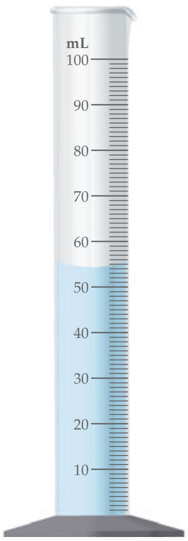
Decimal Notation	Scientific Notation
2,000,000,000	______
______	1.211×10^{9}
0.000874	______
______	3.2×10^{11}

40. Complete the table.

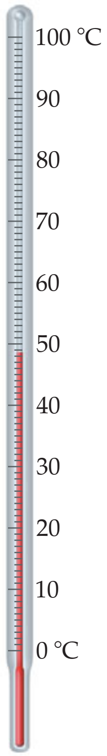
Decimal Notation	Scientific Notation
______	4.2×10^{-3}
315,171,000	______
______	1.8×10^{-11}
1,232,000	______

SIGNIFICANT FIGURES

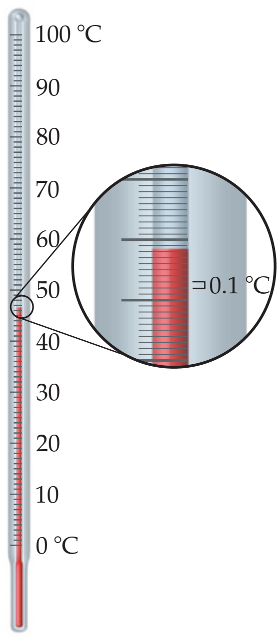
41. Read each instrument to the correct number of significant figures. Laboratory glassware should always be read from the bottom of the *meniscus* (the curved surface at the top of the liquid column).



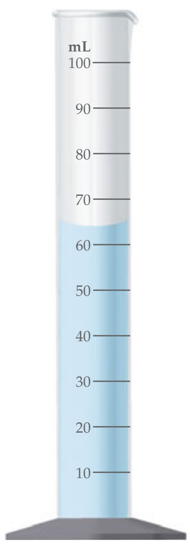
(a)



(b) Celsius

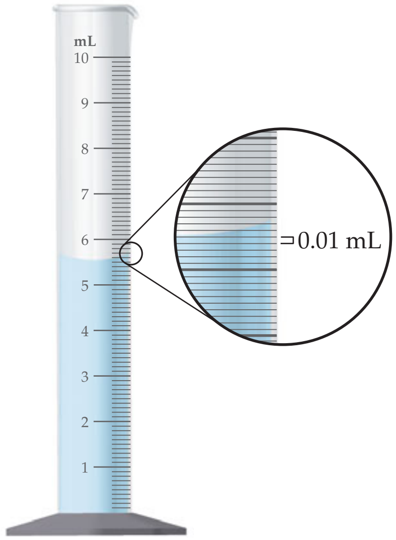


(c) Celsius

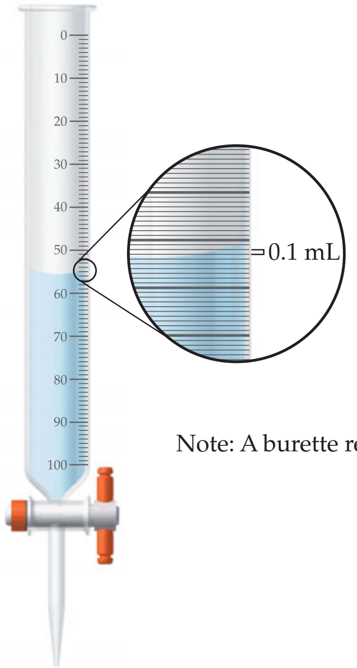


(d)

42. Read each instrument to the correct number of significant figures. Laboratory glassware should always be read from the bottom of the meniscus (the curved surface at the top of the liquid column).



(a)

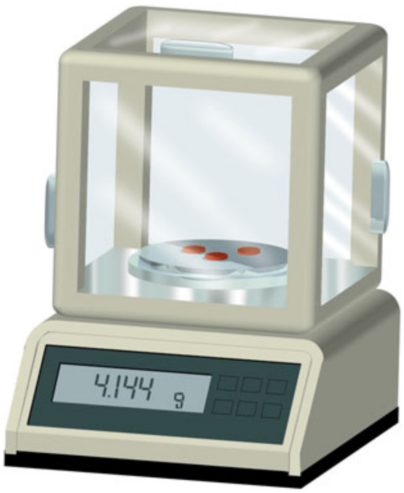


Note: A burette reads from the top down.

(b)

Note: A pipette reads from the top down.

(c)



(d) Note: Digital balances normally display mass to the correct number of significant figures for that particular balance.

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43. For each measured quantity, underline the zeros that are significant and draw an *X* through the zeros that are not.

(a) 0.005050 m
(b) 0.0000000000000060 s
(c) 220,103 kg
(d) 0.00108 in.

44. For each measured quantity, underline the zeros that are significant and draw an *X* through the zeros that are not.

(a) 0.00010320 s
(b) 1,322,600,324 kg
(c) 0.0001240 in.
(d) 0.02061 m

45. How many significant figures are in each measured quantity?
(a) 0.001125 m
(b) 0.1125 m
(c) 1.12500×10^4 m
(d) 11205 m

46. How many significant figures are in each measured quantity?
(a) 13001 kg
(b) 13111 kg
(c) 1.30×10^4 kg
(d) 0.00013 kg

47. Determine whether each of the entries in the table is correct. Correct the entries that are wrong.

Quantity	Significant Figures
(a) 895675 m	6
(b) 0.000869 kg	6
(c) 0.5672100 s	5
(d) 6.022×10^{23} atoms	4

48. Determine whether each of the entries in the table is correct. Correct the entries that are wrong.

Quantity	Significant Figures
(a) 24 days	2
(b) 5.6×10^{-12} s	3
(c) 3.14 m	3
(d) 0.00383 g	5

ROUNDING

49. Round each number to four significant figures.
(a) 255.98612
(b) 0.0004893222
(c) 2.900856×10^{-4}
(d) 2,231,479

50. Round each number to three significant figures.
(a) 10,776.522
(b) 4.999902×10^6
(c) 1.3499999995
(d) 0.0000344988

51. Round each number to two significant figures.
(a) 2.34
(b) 2.35
(c) 2.349
(d) 2.359

52. Round each number to three significant figures.
(a) 65.74
(b) 65.749
(c) 65.75
(d) 65.750

53. Each number was supposed to be rounded to three significant figures. Find the ones that were incorrectly rounded and correct them.
(a) 42.3492 to 42.4
(b) 56.9971 to 57.0
(c) 231.904 to 232
(d) 0.04555 to 0.046

54. Each number was supposed to be rounded to two significant figures. Find the ones that were incorrectly rounded and correct them.
(a) 1.249×10^3 to 1.3×10^3
(b) 3.999×10^2 to 40
(c) 56.21 to 56.2
(d) 0.009964 to 0.010

55. Round the number on the left to the number of significant figures indicated as shown by the example in the first row. (Use scientific notation as needed to avoid ambiguity.)

Number	Rounded to 4 Significant Figures	Rounded to 2 Significant Figures	Rounded to 1 Significant Figure
1.45815	1.458	1.5	1
8.32466			
84.57225			
132.5512			

56. Round the number on the left to the number of significant figures indicated as shown by the example in the first row. (Use scientific notation as needed to avoid ambiguity.)

Number	Rounded to 4 Significant Figures	Rounded to 2 Significant Figures	Rounded to 1 Significant Figure
94.52118	94.52	95	9×10^1
105.4545			
0.455981			
0.009999991			

SIGNIFICANT FIGURES IN CALCULATIONS

57. Perform each calculation to the correct number of significant figures.
(a) $4.5 \times 0.03060 \times 0.391$
(b) $5.55 \div 8.97$
(c) $(7.890 \times 10^{12}) \div (6.7 \times 10^4)$
(d) $67.8 \times 9.8 \div 100.04$

58. Perform each calculation to the correct number of significant figures.
(a) $89.3 \times 77.0 \times 0.08$
(b) $(5.01 \times 10^5) \div (7.8 \times 10^2)$
(c) $4.005 \times 74 \times 0.007$
(d) $453 \div 2.031$

59. Determine whether the answer to each calculation has the correct number of significant figures. If not, correct it.
(a) $34.00 \times 567 \div 4.564 = 4.2239 \times 10^3$
(b) $79.3 \div 0.004 \times 35.4 = 7 \times 10^5$
(c) $89.763 \div 22.4581 = 3.997$
(d) $(4.32 \times 10^{12}) \div (3.1 \times 10^{-4}) = 1.4 \times 10^{16}$

60. Determine whether the answer to each calculation has the correct number of significant figures. If not, correct it.
(a) $45.3254 \times 89.00205 = 4034.05$
(b) $0.00740 \times 45.0901 = 0.334$
(c) $49857 \div 904875 = 0.05510$
(d) $0.009090 \times 6007.2 = 54.605$

61. Perform each calculation to the correct number of significant figures.
(a) $87.6 + 9.888 + 2.3 + 10.77$
(b) $43.7 - 2.341$
(c) $89.6 + 98.33 - 4.674$
(d) $6.99 - 5.772$

62. Perform each calculation to the correct number of significant figures.
(a) $1459.3 + 9.77 + 4.32$
(b) $0.004 + 0.09879$
(c) $432 + 7.3 - 28.523$
(d) $2.4 + 1.777$

63. Determine whether the answer to each calculation has the correct number of significant figures. If not, correct it.
(a) $(3.8 \times 10^5) - (8.45 \times 10^5) = -4.7 \times 10^5$
(b) $0.00456 + 1.0936 = 1.10$
(c) $8475.45 - 34.899 = 8440.55$
(d) $908.87 - 905.34095 = 3.5291$

64. Determine whether the answer to each calculation has the correct number of significant figures. If not, correct it.
(a) $78.9 + 890.43 - 23 = 9.5 \times 10^2$
(b) $9354 - 3489.56 + 34.3 = 5898.74$
(c) $0.00407 + 0.0943 = 0.0984$
(d) $0.00896 - 0.007 = 0.00196$

65. Perform each calculation to the correct number of significant figures.
(a) $(78.4 - 44.889) \div 0.0087$
(b) $(34.6784 \times 5.38) + 445.56$
(c) $(78.7 \times 10^5 \div 88.529) + 356.99$
(d) $(892 \div 986.7) + 5.44$

66. Perform each calculation to the correct number of significant figures.
(a) $(1.7 \times 10^6 \div 2.63 \times 10^5) + 7.33$
(b) $(568.99 - 232.1) \div 5.3$
(c) $(9443 + 45 - 9.9) \times 8.1 \times 10^6$
(d) $(3.14 \times 2.4367) - 2.34$

67. Determine whether the answer to each calculation has the correct number of significant figures. If not, correct it.
(a) $(78.56 - 9.44) \times 45.6 = 3152$
(b) $(8.9 \times 10^5 \div 2.348 \times 10^2) + 121 = 3.9 \times 10^3$
(c) $(45.8 \div 3.2) - 12.3 = 2$
(d) $(4.5 \times 10^3 - 1.53 \times 10^3) \div 34.5 = 86$

68. Determine whether the answer to each calculation has the correct number of significant figures. If not, correct it.
(a) $(908.4 - 3.4) \div 3.52 \times 10^4 = 0.026$
(b) $(1206.7 - 0.904) \times 89 = 1.07 \times 10^5$
(c) $(876.90 + 98.1) \div 56.998 = 17.11$
(d) $(455 \div 407859) + 1.00098 = 1.00210$

UNIT CONVERSION

69. Perform each conversion within the metric system.
(a) 3.55 kg to grams
(b) 8944 mm to meters
(c) 4598 mg to kilograms
(d) 0.0187 L to milliliters

70. Perform each conversion within the metric system.
(a) 155.5 cm to meters
(b) 2491.6 g to kilograms
(c) 248 cm to millimeters
(d) 6781 mL to liters

71. Perform each conversion within the metric system.
(a) 5.88 dL to liters
(b) 3.41×10^{-5} g to micrograms
(c) 1.01×10^{-8} s to nanoseconds
(d) 2.19 pm to meters

72. Perform each conversion within the metric system.
(a) 1.08 Mm to kilometers
(b) 4.88 fs to picoseconds
(c) 7.39×10^{11} m to gigameters
(d) 1.15×10^{-10} m to picometers

73. Perform each conversion between the English and metric systems.
(a) 22.5 in. to centimeters
(b) 126 ft to meters
(c) 825 yd to kilometers
(d) 2.4 in. to millimeters

74. Perform each conversion between the English and metric systems.
(a) 78.3 in. to centimeters
(b) 445 yd to meters
(c) 336 ft to centimeters
(d) 45.3 in. to millimeters

75. Perform each conversion between the metric and English systems.
(a) 40.0 cm to inches
(b) 27.8 m to feet
(c) 10.0 km to miles
(d) 3845 kg to pounds

76. Perform each conversion between the metric and English systems.
(a) 254 cm to inches
(b) 89 mm to inches
(c) 7.5 L to quarts
(d) 122 kg to pounds

77. Complete the table:

m	km	Mm	Gm	Tm
5.08×10^8 m	______	508 Mm	______	______
______	______	27,976 Mm	______	______
______	______	______	______	1.77 Tm
______	1.5×10^5 km	______	______	______
______	______	______	423 Gm	______

78. Complete the table:

s	ms	μs	ns	ps
1.31×10^{-4} s	______	131 μs	______	______
______	______	______	______	12.6 ps
______	______	______	155 ns	______
______	1.99×10^{-3} ms	______	______	______
______	______	8.66×10^{-5} μs	______	______

79. Convert 2.255×10^{10} g to each unit:
(a) kg
(b) Mg
(c) mg
(d) metric tons (1 metric ton = 1000 kg)

80. Convert 1.88×10^{-6} g to each unit.
(a) mg
(b) cg
(c) ng
(d) μg

81. A student loses 3.3 lb in one month. How many grams did he lose?

82. A student gains 1.9 lb in two weeks. How many grams did he gain?

83. A runner wants to run 10.0 km. She knows that her running pace is 7.5 mi/h. How many minutes must she run? *Hint:* Use 7.5 mi/h as a conversion factor between distance and time.

84. A cyclist rides at an average speed of 24 mi/h. If she wants to bike 195 km, how long (in hours) must she ride?

85. A recipe calls for 5.0 qt of milk. What is this quantity in cubic centimeters?

86. A gas can holds 2.0 gal of gasoline. What is this quantity in cubic centimeters?

UNITS RAISED TO A POWER

87. Fill in the blanks.
(a) $1.0\ \text{km}^2 = $ ______ m^2
(b) $1.0\ \text{cm}^3 = $ ______ m^3
(c) $1.0\ \text{mm}^3 = $ ______ m^3

88. Fill in the blanks.
(a) $1.0\ \text{ft}^2 = $ ______ in.^2
(b) $1.0\ \text{yd}^2 = $ ______ ft^2
(c) $1.0\ \text{m}^2 = $ ______ yd^2

89. The hydrogen atom has a volume of approximately $6.2 \times 10^{-31}\ \text{m}^3$. What is this volume in each unit?
(a) cubic picometers
(b) cubic nanometers
(c) cubic angstroms ($1\ \text{angstrom} = 10^{-10}\ \text{m}$)

90. Earth has a surface area of 197 million square miles. What is its area in each unit?
(a) square kilometers
(b) square megameters
(c) square decimeters

91. A modest-sized house has an area of 215 m^2. What is its area in each unit?
(a) km^2
(b) dm^2
(c) cm^2

92. A classroom has a volume of 285 m^3. What is its volume in each unit?
(a) km^3
(b) dm^3
(c) cm^3

93. Total U.S. farmland occupies 954 million acres. How many square miles is this?
($1\ \text{acre} = 43{,}560\ \text{ft}^2$; $1\ \text{mi} = 5280\ \text{ft}$)

94. The average U.S. farm occupies 435 acres. How many square miles is this?
($1\ \text{acre} = 43{,}560\ \text{ft}^2$; $1\ \text{mi} = 5280\ \text{ft}$)

DENSITY

95. A sample of an unknown metal has a mass of 35.4 g and a volume of 3.11 cm^3. Calculate its density and identify the metal by comparison to Table 2.4.

96. A new penny has a mass of 2.49 g and a volume of 0.349 cm^3. Is the penny pure copper?

97. Glycerol is a syrupy liquid often used in cosmetics and soaps. A 2.50-L sample of pure glycerol has a mass of 3.15×10^3 g. What is the density of glycerol in grams per cubic centimeter?

98. An aluminum engine block has a volume of 4.77 L and a mass of 12.88 kg. What is the density of the aluminum in grams per cubic centimeter?

99. A supposedly gold tooth crown is tested to determine its density. It displaces 10.7 mL of water and has a mass of 206 g. Could the crown be made of gold?

100. A vase is said to be solid platinum. It displaces 18.65 mL of water and has a mass of 157 g. Could the vase be solid platinum?

101. Ethylene glycol (antifreeze) has a density of 1.11 g/cm^3.
- **(a)** What is the mass in grams of 387 mL of this liquid?
- **(b)** What is the volume in liters of 3.46 kg of this liquid?

102. Acetone (fingernail-polish remover) has a density of 0.7857 g/cm^3.
- **(a)** What is the mass in grams of 17.56 mL of acetone?
- **(b)** What is the volume in milliliters of 7.22 g of acetone?

CUMULATIVE PROBLEMS

103. A thief uses a bag of sand to replace a gold statue that sits on a weight-sensitive, alarmed pedestal. The bag of sand and the statue have exactly the same volume, 1.75 L. (Assume that the mass of the bag is negligible.)
- **(a)** Calculate the mass of each object. (density of gold $= 19.3\,g/cm^3$; density of sand $= 3.00\,g/cm^3$)
- **(b)** Did the thief set off the alarm? Explain.

104. One of the particles that composes an atom is the proton. A proton has a radius of approximately 1.0×10^{-13} cm and a mass of 1.7×10^{-24} g. Determine the density of a proton.

$$\left(\text{volume of a sphere} = \frac{4}{3}\pi r^3;\ \pi = 3.14\right)$$

105. A block of metal has a volume of 13.4 $in.^3$ and weighs 5.14 lb. What is its density in grams per cubic centimeter?

106. A log is either oak or pine. It displaces 2.7 gal of water and weighs 19.8 lb. Is the log oak or pine? (density of oak $= 0.9\,g/cm^3$; density of pine $= 0.4\,g/cm^3$)

107. The density of aluminum is 2.7 g/cm^3. What is its density in kilograms per cubic meter?

108. The density of platinum is 21.4 g/cm^3. What is its density in pounds per cubic inch?

109. A typical backyard swimming pool holds 150 yd^3 of water. What is the mass in pounds of the water?

110. An iceberg has a volume of 8975 ft^3. What is the mass in kilograms of the iceberg?

111. The mass of fuel in an airplane must be carefully accounted for before takeoff. If a 747 contains 155,211 L of fuel, what is the mass of the fuel in kilograms? Assume the density of the fuel to be 0.768 g/cm^3.

112. A backpacker carries 2.5 L of white gas as fuel for her stove. How many pounds does the fuel add to her load? Assume the density of white gas to be 0.79 g/cm^3.

113. Honda produces a hybrid electric car called the Honda Insight. The Insight has both a gasoline-powered engine and an electric motor and has an EPA gas mileage rating of 43 miles per gallon on the highway. What is the Insight's rating in kilometers per liter?

114. You rent a car in Germany with a gas mileage rating of 12.8 km/L. What is its rating in miles per gallon?

115. A car has a mileage rating of 38 miles per gallon of gasoline. How many miles can the car travel on 76.5 liters of gasoline?

116. A hybrid SUV consumes fuel at a rate of 12.8 km/L. How many miles can the car travel on 22.5 gallons of gasoline?

117. Block A of an unknown metal has a volume of 125 cm^3. Block B of a different metal has a volume of 145 cm^3. If block A has a greater mass than block B, what can be said of the relative densities of the two metals? (Assume that both blocks are solid.)

118. Block A of an unknown metal has a volume of 125 cm^3. Block B of a different metal has a volume of 105 cm^3. If block A has a greater mass than block B, what can be said of the relative densities of the two metals? (Assume that both blocks are solid.)

119. The masses and volumes of two cylinders are measured. The mass of cylinder 1 is 1.35 times the mass of cylinder 2. The volume of cylinder 1 is 0.792 times the volume of cylinder 2. If the density of cylinder 1 is 3.85 g/cm^3, what is the density of cylinder 2?

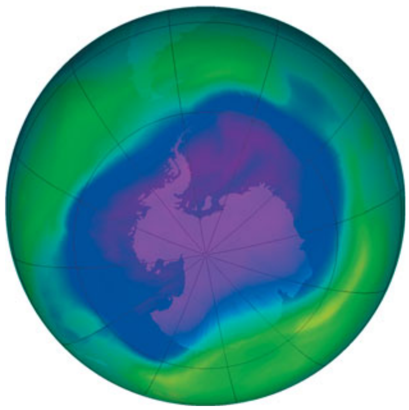
120. A bag contains a mixture of copper and lead BBs. The average density of the BBs is 9.87 g/cm^3. Assuming that the copper and lead are pure, determine the relative amounts of each kind of BB.

HIGHLIGHT PROBLEMS

121. In 1999, NASA lost a $94 million orbiter because one group of engineers used metric units in their calculations while another group used English units. Consequently, the orbiter descended too far into the Martian atmosphere and burned up. Suppose that the orbiter was to have established orbit at 155 km and that one group of engineers specified this distance as 1.55×10^5 m. Suppose further that a second group of engineers programmed the orbiter to go to 1.55×10^5 ft. What was the difference in kilometers between the two altitudes? How low did the probe go?

▲ The $94 million Mars Climate Orbiter was lost in the Martian atmosphere in 1999 because two groups of engineers failed to communicate to each other the units that they used in their calculations.

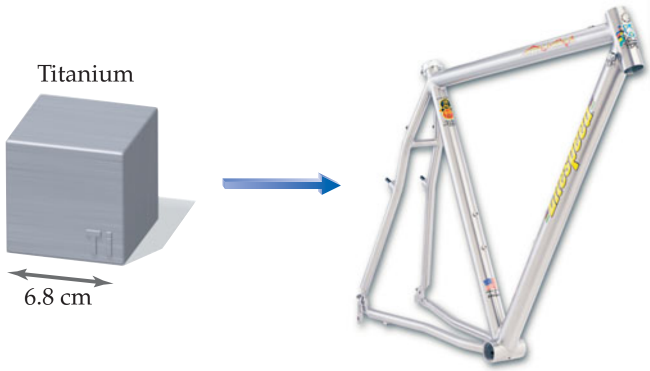
122. A NASA satellite showed that in 2009 the ozone hole over Antarctica had a maximum surface area of 24.1 million km^2. The largest ozone hole on record occurred in 2006 and had a surface area of 29.6 million km^2. Calculate the difference in diameter (in meters) between the ozone hole in 2009 and in 2006.



▲ A layer of ozone gas (a form of oxygen) in the upper atmosphere protects Earth from harmful ultraviolet radiation in sunlight. Human-made chemicals react with the ozone and deplete it, especially over the Antarctic at certain times of the year (the so-called ozone hole). The region of low ozone concentration in 2006 (represented here by the dark purple color) was the largest on record.

123. In 1999, scientists discovered a new class of black holes with masses 100 to 10,000 times the mass of our sun, but occupying less space than our moon. Suppose that one of these black holes has a mass of 1×10^3 suns and a radius equal to one-half the radius of our moon. What is its density in grams per cubic centimeter? The mass of the sun is 2.0×10^{30} kg, and the radius of the moon is 2.16×10^3 mi. (Volume of a sphere $= \frac{4}{3}\pi r^3$.)

124. A titanium bicycle frame contains the same amount of titanium as a titanium cube measuring 6.8 cm on a side. Use the density of titanium to calculate the mass in kilograms of titanium in the frame. What would be the mass of a similar frame composed of iron?



▲ A titanium bicycle frame contains the same amount of titanium as a titanium cube measuring 6.8 cm on a side.

▸ANSWERS TO SKILLBUILDER EXERCISES

Skillbuilder 2.1 $\$1.2102 \times 10^{13}$

Skillbuilder 2.2 3.8×10^{-5}

Skillbuilder 2.3 103.4 °F

Skillbuilder 2.4
- **(a)** four significant figures
- **(b)** three significant figures
- **(c)** two significant figures
- **(d)** unlimited significant figures
- **(e)** three significant figures
- **(f)** ambiguous

Skillbuilder 2.5
- **(a)** 0.001 or 1×10^{-3}
- **(b)** 0.204

Skillbuilder 2.6
- **(a)** 7.6
- **(b)** 131.11

Skillbuilder 2.7
- **(a)** 1288
- **(b)** 3.12

Skillbuilder 2.8 22.0 in.

Skillbuilder 2.9 5.678 km

Skillbuilder 2.10 0.28 L

Skillbuilder 2.11 46.6 laps

Skillbuilder Plus, p. 31 1.06×10^4 m

Skillbuilder 2.12 4747 cm^3

Skillbuilder 2.13 1.52×10^5 $in.^3$

Skillbuilder 2.14 Yes, the density is 21.4 g/cm^3 and matches that of platinum.

Skillbuilder 2.15 4.4×10^{-2} cm^3

Skillbuilder Plus, p. 37 1.95 kg

Skillbuilder 2.16 16 kg

Skillbuilder 2.17 $d = 19.3\,g/cm^3$; yes, the density is consistent with that of gold.

▸ANSWERS TO CONCEPTUAL CHECKPOINTS

2.1 **(c)** Multiplying by 10^{-3} is equivalent to moving the decimal point three places to the left.

2.2 **(b)** The last digit is considered to be uncertain by ±1.

2.3 **(b)** The result of the calculation in **(a)** would be reported as 4; the result of the calculation in **(b)** would be reported as 1.5.

2.4 **(d)** The diameter would be expressed as 28 nm.

2.5 **(c)** Kilometers must appear in the numerator and meters in the denominator, and the conversion factor in **(d)** is incorrect ($10^3\,km \neq 1\,m$).

2.6 **(d)** $(3\text{ ft}) \times (3\text{ ft}) \times (3\text{ ft}) = 27\text{ ft}^3$